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Exponential stability of nonlinear impulsive neutral integro-differential equations[☆]

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Abstract

In this paper, a nonlinear impulsive neutral integro-differential equation with time-varying delays is considered. By establishing a singular impulsive delay integro-differential inequality and transforming the n-dimensional impulsive neutral integro-differential equation to a $2n$-dimensional singular impulsive delay integro-differential equation, some sufficient conditions ensuring the global exponential stability in PC^1 of the zero solution of an impulsive neutral integro-differential equation are obtained. The results extend and improve the earlier publications. An example is also discussed to illustrate the efficiency of the obtained results.

Keywords: Neutral; Impulsive; Singular impulsive integro-differential inequality; Exponential stability; Delay

1. Introduction

Integro-differential equations have been extensively studied over the past few decades and have found many applications in a variety of areas, where it is necessary to take into account aftereffect or delay such as control theory, biology, ecology, medicine, etc. (cf. [5,8,13]). Especially, one always describes a model which possesses hereditary properties by integro-differential equations in practice.

As is well-known, stability is one of the major problems encountered in applications, and has received considerable attention due to its important role in the applications. In particular, delay effect on the stability and other behaviors of integro-differential equations have been widely studied in the literature (for instance, see [14,16,24,32,33,37]). However, besides delay effect, impulsive effect likewise exists in a wide variety of evolutionary processes that are subject to sudden changes in their states. In recent years, the interest of researchers on impulsive integro-differential equations has grown very fast and a huge mass of interesting results on impulsive integro-differential equations have been reported, (cf. [1,2,4,7,15,21,25–27,39]). However, to the best of our knowledge, there are no results on the problems of the exponential stability of solutions for nonlinear impulsive neutral integro-differential equations due to some theoretical and technical difficulties. Based on this, this article is devoted to the discussion of this problem.

As the paper [28] pointed out, differential inequalities are very important tools for investigating the stability of impulsive differential equations, see [6,12,18,27–29]. In [28], Xu et al. established the singular impulsive delay

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differential inequality and transformed the n-dimensional impulsive neutral differential equation to a $2n$-dimensional singular impulsive delay differential equation and derived some sufficient conditions ensuring the global exponential stability in the space PC^1 of the zero solution of a nonlinear impulsive neutral differential equation with time-varying delays. In [27], Xu et al. established the delay integro-differential inequality with impulsive initial conditions and derived some sufficient conditions ensuring the global exponential stability of solutions for the impulsive integro-differential equations. In this paper, we will improve the inequality established in [27] such that it is effective for impulsive neutral integro-differential equations. By establishing a singular impulsive delay integro-differential inequality and using the methods in [28], some sufficient conditions ensuring the global exponential stability in the space PC^1 of the zero solution of a nonlinear impulsive neutral integro-differential equation with time-varying delays are obtained. The results extend and improve the earlier publications. An example is given to illustrate the theory.

2. Model and preliminaries

Let E denote the n-dimensional unit matrix, $\mathcal{N} \triangleq \{1, 2, \ldots, n\}$. For $A, B \in R^{n\times n}$ or $A, B \in R^n$, $A \geq B (A \leq B, A > B, A < B)$ means that each pair of corresponding elements of A and B satisfies the inequality "$\geq (\leq, >, <)$". Especially, A is called a nonnegative matrix if $A \geq 0$, and z is called a positive vector if $z > 0$.

$C[X, Y]$ denote the space of continuous mappings from the topological space X to the topological space Y. Especially, let $C \triangleq C[(-\infty, 0], R^n]$.

$$PC[J, R^n] = \{\psi : J \to R^n | \psi(s) \text{ is continuous for all but at most countable points } s \in J \text{ and at these points } s \in J, \psi(s^+) \text{ and } \psi(s^-) \text{ exist}, \psi(s) = \psi(s^+)\},$$

where $J \subset R$ is an interval, $\psi(s^+)$ and $\psi(s^-)$ denote the right-hand and left-hand limits of the function $\psi(s)$, respectively. Especially, let $PC \triangleq PC[(-\infty, 0], R^n]$.

$$PC^1[J, R^n] = \{\psi : J \to R^n | \psi(s) \text{ is continuously differentiable for all but at most countable points } s \in J \text{ and at these points } s \in J, \psi(s^+), \psi(s^-), \psi'(s^+) \text{ and } \psi'(s^-) \text{ exist}, \psi(s) = \psi(s^+), \psi'(s) \triangleq \psi'(s^+)\},$$

where $\psi'(s)$ denote the derivative of $\psi(s)$. Especially, let $PC^1 \triangleq PC^1[(-\infty, 0], R^n]$.

$$L^e = \left\{\psi(s) : R^+ \to R,\ \text{where } R^+ = [0, \infty) | \psi(s) \text{ is piecewise continuous and satisfies } \int_0^\infty e^{\lambda_0 s} |\psi(s)| ds < \infty,\ \text{where } \lambda_0 > 0 \text{ is constant}\right\}.$$

For $x \in R^n$, $A \in R^{n\times n}$, we define

$$[x]^+ = \text{col}\{|x_i|\} = (|x_1|, \ldots, |x_n|)^T, \qquad [A]^+ = (|a_{ij}|)_{n\times n}.$$

For $\varphi(t) \in C[J, R^n]$ or $\varphi(t) \in PC[J, R^n]$, we define

$$[\varphi(t)]_\tau = \text{col}([\varphi_i(t)]_\tau), \qquad [\varphi(t)]_\tau^+ = [[\varphi(t)]^+]_\tau, \qquad [\varphi_i(t)]_\tau = \sup_{-\tau\leq s<0} \{\varphi_i(t+s)\}, \quad i \in \mathcal{N},$$

$$[\varphi(t)]_\infty = \text{col}([\varphi_i(t)]_\infty), \qquad [\varphi(t)]_\infty^+ = [[\varphi(t)]^+]_\infty, \qquad [\varphi_i(t)]_\infty = \sup_{-\infty<s\leq 0} \{\varphi_i(t+s)\}, \quad i \in \mathcal{N},$$

and $D^+\varphi(t)$ denotes the upper right derivative of $\varphi(t)$ at time t.

For $\varphi(t) \in C$ or $\varphi(t) \in PC$, we introduce the following norm:

$$\|\varphi\|_\infty = \max_{1\leq i\leq n} \{[\varphi_i(s)]_\infty^+\}.$$

For $\varphi \in PC^1$, we introduce the following norm:

$$\|\varphi\|_{1\infty} = \max_{1\leq i\leq n} \{[\varphi_i(s)]_\infty^+, [\varphi_i'(s)]_\infty^+\}.$$