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An exact solution of start-up flow for the fractional generalized Burgers' fluid in a porous half-space

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Abstract

Modified Darcy's law for fractional generalized Burgers' fluid in a porous medium is introduced. The flow near a wall suddenly set in motion for a fractional generalized Burgers' fluid in a porous half-space is investigated. The velocity of the flow is described by fractional partial differential equations. By using the Fourier sine transform and the fractional Laplace transform, an exact solution of the velocity distribution is obtained. Some previous and classical results can be recovered from our results, such as the velocity solutions of the Stokes' first problem for viscous Newtonian, second grade, Maxwell, Oldroyd-B or Burgers' fluids.

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1. Introduction

There are very few cases in which the exact analytical solution of Navier–Stokes equations can be obtained. These are even rare if the constitutive equations for viscoelastic fluids are considered, because it is difficult to suggest a single model which exhibits all properties of viscoelastic fluids as is done for the Newtonian fluids. For this reason many models of constitutive equations have been proposed. Very little efforts [1–14] have so far been made to discuss the flows of viscoelastic fluids with a fractional calculus approach. This approach is a natural tool in describing complex dynamical systems such as polymeric materials. The large number of polymer chain configurations gives rise to the complexity in the configurational dynamics of these materials. This is reflected in the intermittency in the chain segment motions and in the tendency for the particles to cluster and thus move in a collective fashion.

The motion of a fluid caused by the sudden motion of a plate from rest, also named as Stokes' first problem or Rayleigh's problem [15], not only is of fundamental theoretical interest but also occurs in many applied problems.

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This problem provides insights into the analysis of boundary layer growth and flow bounded by an infinite flat plate. By using similarity transformation, Stokes [15] has given the solution of the problem for a viscous fluid. But in a non-Newtonian fluid, such transformation does not work. Due to the variety of fluids, various researchers extended the Stokes' problem with different conditions and fluid models, for example, Teipel [16] for a second grade fluid, Morrison [17] for a Jeffrey fluid with no small viscosity, Huilgol [18] for a fluid with small viscosity, Tanner [19] for a viscoelastic fluid, Preziosi and Joseph [20] for viscoelastic fluids with a viscosity and relaxation kernel, Phan-Thien and Chew [21] for a modified Phan-Thien–Tanner model and for Rivlin–Fricksen fluids [8,9,22–24].

On the other hand, the study of viscoelastic fluids through porous media has gained much importance in view of their promising applications in biorheology and engineering fields such as enhanced oil recovery, paper and textile coating, and composite manufacturing processes. Jordan et al. [25,26] discussed the Stokes' first problem in a porous medium using Laplace transform treatment. Their solution does not satisfy the initial condition. Also, they have used the Darcy's law for a Newtonian fluid flow in order to describe the effects of the pores on the velocity of a non-Newtonian fluid which in fact is of argument. More recently, Tan and Masuoka [27–30] developed a modified Darcy–Brinkman model and investigated the Stokes' first problem for viscoelastic fluids in a porous half-space. Based on their works, in this paper, we extend the Stokes' first problem for a Burgers' fluid in a porous half-space with fractional derivative model. An exact solution of the velocity distribution was obtained by using Fourier sine transform and fractional Laplace transform. Some previous and classical results can be recovered from our results, such as the velocity solutions of the Stokes' first problem for a viscous Newtonian, second grade, Maxwell, Oldroyd-B, Burgers' fluid or generalized Burgers' fluid.

2. Governing equations

The Cauchy stress tensor T in a fractional generalized Burgers' fluid is

$$T = -pI + S \tag{1}$$

$$(1 + \lambda_1 \tilde{D}_t^{\alpha} + \lambda_2 \tilde{D}_t^{2\alpha})S = \mu(1 + \lambda_3 \tilde{D}_t^{\beta} + \lambda_4 \tilde{D}_t^{2\beta})A, \tag{2}$$

where $-pI$ indicates the indeterminate spherical stress, $A = L + L^T$ is first Rivlin–Ericksen tensor, μ is the dynamic viscosity, L is the velocity gradient, λ_1 and λ_3 ($<\lambda_1$) are the relaxation and retardation times, S is the extra stresstensor, λ_2, λ_4 are material constants, α, β are fractional coefficients and $0 \le \alpha, \beta \le 1$. $\tilde{D}_t$ is the upper convected time derivative defined by [5]

$$\tilde{D}_t^{\alpha} S = D_t^{\alpha} S - LS - SL^T, \tag{3}$$

where D_t^{α} is the fractional material derivative, which is defined as [31]

$$_aD_t^{\alpha}[f(t)] = \frac{1}{\Gamma(k-\alpha)} \frac{\mathrm{d}^k}{\mathrm{d}t^k} \int_a^t (t-\tau)^{k-\alpha-1} f(\tau)\mathrm{d}\tau \quad (k - 1 \le \alpha < k), \tag{4}$$

where $\Gamma(\cdot)$ is the gamma function, and

$$\tilde{D}_t^{2\alpha} S = \tilde{D}_t^{\alpha}(\tilde{D}_t^{\alpha} S). \tag{5}$$

The flows under consideration have the following velocity field

$$V = u(y, t)i, \tag{6}$$

where i and u are the unit vector and velocity parallel to the x-axis, respectively.

The velocity field (6) automatically satisfies the incompressibility condition. Since u is a function of y and t, the stress field will also depend upon y and t. Now, Eq. (2) together with the initial condition (the fluid being at rest up to the moment $t = 0$)

$$S(y, 0) = 0 \tag{7}$$