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Shock reflection for general quasilinear hyperbolic systems of conservation laws

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Abstract

This paper concerns the reflection of shock waves for general quasilinear hyperbolic systems of conservation laws in one space dimension. It is shown that the mixed initial–boundary value problem for general quasilinear hyperbolic systems of conservation laws with nonlinear boundary conditions on the quarter-plane $\{(t, x) \mid t \geq 0, x \geq 0\}$ admits a unique global piecewise C^1 solution $u = u(t, x)$ containing only shock waves with small amplitude and this solution possesses a global structure similar to that of the Riemann solution $u = U(\frac{x}{t})$ of the corresponding Riemann problem, if the positive eigenvalues are genuinely nonlinear and the Riemann solution has only shock waves, and no rarefaction waves and contact discontinuities. Our result indicates that the Riemann solution $u = U(\frac{x}{t})$ consisting of only shock waves possesses a semi-global nonlinear structure stability.

MSC: 35L65; 35L50; 35L67

Keywords: Mixed initial–boundary value problem; Quasilinear hyperbolic systems of conservation laws; Genuinely nonlinear; Shock wave

1. Introduction and main result

Consider the following quasilinear hyperbolic system of conservation laws:

$$\frac{\partial u}{\partial t} + \frac{\partial f(u)}{\partial x} = 0, \qquad u = u(t, x) \in \mathcal{U} \subset \mathbf{R}^n, t > 0, \tag{1.1}$$

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where $u = (u_1, \ldots, u_n)^T$ is the unknown vector function of (t, x), $f : \mathcal{U} \to \mathbf{R}^n$ is a given C^3 vector function of u.

It is assumed that system (1.1) is strictly hyperbolic, i.e., for any given u on the domain under consideration, the Jacobian $A(u) = \nabla f(u)$ has n real distinct eigenvalues

$$\lambda_1(u) < \lambda_2(u) < \cdots < \lambda_n(u). \tag{1.2}$$

Let $l_i(u) = (l_{i1}(u), \ldots, l_{in}(u))$ (resp. $r_i(u) = (r_{i1}(u), \ldots, r_{in}(u))^T$) be a left (resp. right) eigenvector corresponding to $\lambda_i(u)$ $(i = 1, \ldots, n)$:

$$l_i(u)A(u) = \lambda_i(u)l_i(u) \qquad (\text{resp.} A(u)r_i(u) = \lambda_i(u)r_i(u)), \tag{1.3}$$

we have

$$\det|l_{ij}(u)| \neq 0 \qquad (\text{equivalently, } \det|r_{ij}(u)| \neq 0). \tag{1.4}$$

Without loss of generality, we may assume that on the domain under consideration

$$l_i(u)r_j(u) \equiv \delta_{ij} \qquad (i, j = 1, \ldots, n) \tag{1.5}$$

and

$$r_i^T(u)r_i(u) \equiv 1 \qquad (i = 1, \ldots, n), \tag{1.6}$$

where δ_{ij} stands for the Kronecker symbol.

Clearly, all $\lambda_i(u)$, $l_{ij}(u)$ and $r_{ij}(u)$ $(i, j = 1, \ldots, n)$ have the same regularity as $A(u)$, i.e., C^2 regularity.

We also assume that on the domain under consideration, each characteristic field is either genuinely nonlinear (g.n.l.) or linearly degenerate (l.d.g.) in the sense of Lax [1]:

$$\nabla\lambda_i(u)r_i(u) \neq 0 (\text{g.n.l.}), \qquad \nabla\lambda_i(u)r_i(u) \equiv 0 (\text{l.d.g.}). \tag{1.7}$$

We are interested in solutions taking values in a small neighborhood of a given state in $\mathbf{R}^n$ and, without loss of generality, we can choose this set to be the ball $\mathcal{U} := \mathbf{B}(\varepsilon)$ centered at the origin with suitably small radius ε. We first recall that the Riemann problem for system (1.1) is a special Cauchy problem with the piecewise constant initial data

$$t = 0 : u = \begin{cases} u_L, & x < 0, \\ u_R, & x > 0, \end{cases} \tag{1.8}$$

where u_L and u_R are constant states in $\mathcal{U}$. It is well known that the Riemann problem (1.1) and (1.8) has a unique self-similar solution composed of $n + 1$ constant states separated by shock waves, rarefaction waves, and contact discontinuities (they are called elementary waves) provided that the states are in a small neighborhood of a given state (see [1] or [5]). In the following, the set $\mathcal{U}$ is chosen such that the Riemann problem is always well-posed in this sense.

We assume that on the domain under consideration, the eigenvalues of $A(u) = \nabla f(u)$ satisfy the non-characteristic condition

$$\lambda_r(u) < 0 < \lambda_s(u) \qquad (r = 1, \ldots, m; s = m + 1, \ldots, n). \tag{1.9}$$

We are concerned with the global existence of piecewise C^1 solutions containing only shock waves to the mixed initial–boundary value problem for system (1.1) on the domain

$$D = \{(t, x) \mid t \geq 0, x \geq 0\} \tag{1.10}$$