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Finite dimensionality of the global attractors for von Karman equations with nonlinear interior dissipation

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Abstract

In this paper we study the global attractors for von Karman equations with nonlinear interior dissipation. We prove regularity and then establish finite dimensionality of the global attractors without assuming large values for the damping parameter.

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1. Introduction

The main objective of this paper is to study the regularity and finite dimensionality of the global attractor for the following von Karman system:

$$w_{tt} + \Delta^2 w + g(w_t) = [\mathcal{F}(w), w] + h \qquad \text{in } (0, +\infty) \times \Omega \tag{1.1}$$

$$\Delta^2 \mathcal{F}(w) = -[w, w] \qquad \text{in } (0, +\infty) \times \Omega \tag{1.2}$$

$$w = \frac{\partial w}{\partial \nu} = \mathcal{F} = \frac{\partial \mathcal{F}}{\partial \nu} = 0 \qquad \text{on } (0, +\infty) \times \partial\Omega \tag{1.3}$$

$$w(0, \cdot) = w_0, \qquad w_t(0, \cdot) = w_1 \qquad \text{in } \Omega \tag{1.4}$$

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where Ω is a bounded smooth domain in R^2, the vector ν denotes an outward normal, $h \in L_2(\Omega)$ and the von Karman bracket is given by

$$[u, v] \equiv \frac{\partial^2 u}{\partial x_1^2}\frac{\partial^2 v}{\partial x_2^2} + \frac{\partial^2 u}{\partial x_2^2}\frac{\partial^2 v}{\partial x_1^2} - 2\frac{\partial^2 u}{\partial x_1 \partial x_2}\frac{\partial^2 v}{\partial x_1 \partial x_2}.$$

The damping function $g \in C^1(R)$ satisfies the conditions

$$g(0) = 0, \qquad 0 < m \le \acute{g}(s), \qquad \forall\, s \in R \tag{1.5}$$

and

$$\acute{g}(s) \le M(1 + sg(s)), \qquad \forall\, s \in R. \tag{1.6}$$

In the case where $g(\cdot)$ is linear, the weak attractors for (1.1)–(1.4) were studied in [1], and the existence and finite dimensionality of the weak attractors were established for large values of the damping parameter. The well-posedness of weak solutions of problem (1.1)–(1.4) has been established (see [2,3]) by using the sharp regularity of Airy's stress function obtained in [4]. In the case of nonlinear dissipation, the global attractors for the problem (1.1)–(1.4) were investigated in [2,3] and references therein. In these articles the existence and finite dimensionality of attractors have also been proved for large values of the damping parameter. Recently in [5] the existence of a global attractor for (1.1)–(1.4) is shown without assuming large values of the damping parameter.

Our main goal in this paper is to prove regularity and then establish finite dimensionality of the global attractor for the problem (1.1)–(1.4) without assuming large values for the damping parameter.

2. Preliminaries

Denote the spaces $\overset{\circ}{W}{}_2^s(\Omega)$, $W_2^s(\Omega)$ and $L_2(\Omega)$ by H_0^s, H^s, and H respectively. The scalar product and norm in H are denoted by $\langle , \rangle$ and $\|\cdot\|$. We also denote the norm in H^s by $\|\cdot\|_s$ and introduce the spaces $\mathcal{H} = H_0^2 \times H$ and $\mathcal{H}^1 = (H^4 \cap H_0^2) \times H_0^2$. As mentioned above it is known that under condition (1.5) the solution operator $S(t)(w_0, w_1) = (w(t), w_t(t))$, $t \ge 0$, of problem (1.1)–(1.4) generates a C^0-semigroup on the energy space $\mathcal{H}$ (see [2,3]) in which

$$\begin{aligned} &E(w(t) - v(t)) + \int_s^t \int_\Omega (g(w_t(\tau, x)) - g(v_t(\tau, x)))(w_t(\tau, x) - v_t(\tau, x))\mathrm{d}x\mathrm{d}\tau \\ &\quad \le E(w(s) - v(s)) + \int_s^t \langle [\mathcal{F}(w(\tau)), w(\tau)] - [\mathcal{F}(v(\tau)), v(\tau)], w_t(\tau) - v_t(\tau)\rangle \mathrm{d}\tau, \end{aligned} \tag{2.1}$$

holds for $(w(t), w_t(t)) = S(t)(w_0, w_1)$ and $(v(t), v_t(t)) = S(t)(v_0, v_1)$, where $E(u(t)) = \frac{1}{2}(\|\Delta u(t)\|^2 + \|u_t(t)\|^2)$ and $t \ge s \ge 0$.

Let $\mathcal{N}$ be the set of fixed points of $S(t)$, i.e. $\mathcal{N}$ consists of the points $(w, 0)$, where $w = w(x)$ is a solution of the problem

$$\begin{aligned} &\Delta^2 w = [\mathcal{F}(w), w] + h \qquad \text{in } \Omega, \\ &\Delta^2 \mathcal{F}(w) = -[w, w] \qquad \text{in } \Omega, \\ &w = \frac{\partial w}{\partial \nu} = \mathcal{F} = \frac{\partial \mathcal{F}}{\partial \nu} = 0 \qquad \text{on } \partial\Omega. \end{aligned}$$

It is known that $\mathcal{N}$ is bounded in $\mathcal{H}^1$.