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Existence and global attractivity of unique positive periodic solution for a Lasota–Wazewska model☆

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Abstract

In this paper we consider the Lasota–Wazewska model

$$x'(t) = -a(t)x(t) + \sum_{i=1}^{m} p_i(t)\mathrm{e}^{-q_i(t)x(t-\tau_i(t))}.$$

By using a fixed point theorem, some criteria are established for the existence of the unique positive ω-periodic solution $\tilde{x}$ of the above equation. In particular, we not only give the conclusion of convergence of x_n to $\tilde{x}$, where $\{x_n\}$ is a successive sequence, but also show that $\tilde{x}$ is a global attractor of all other positive solutions.

MSC: 34K15

Keywords: Periodic solution; Global attractivity; Lasota–Wazewska model; Uniqueness

1. Introduction

Functional differential equations with periodic delays appear in some ecological models, for example, the model of dynamic disease [14] and the model of the survival of red blood cells in an animal [18], and so on. For these equations, one of the important properties is

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whether they can support positive periodic solutions. In addition, the asymptotic behavior of periodic solutions is also very important. In recent years, periodic population dynamics has become a very popular subject. Several different periodic models have been studied by many authors. See [1,3,6–13,15–17,19] and references therein.

In this paper, we shall study the existence and global attractivity of unique positive periodic solution of the following generalized Lasota–Wazewska model

$$x'(t) = -a(t)x(t) + \sum_{i=1}^{m} p_i(t)\mathrm{e}^{-q_i(t)x(t-\tau_i(t))}, \quad t \geqslant 0. \tag{1}$$

Some special cases of Eq. (1) have been investigated. For example, the delay differential equation

$$x'(t) = -\alpha x(t) + \beta \mathrm{e}^{-\gamma x(t-\tau)}, \quad t \geqslant 0, \tag{2}$$

where α, β, γ and τ are positive constants, was used by Wazewska–Czyzewska and Lasota [18] as a model for the survival of red blood cells in an animal. The oscillation and global attractivity of Eq. (2) have been studied by Kulenovic and Ladas [11] and by Kulenovic et al. [12], respectively. For further investigation in this area, for example, the delay differential equations

$$x'(t) = -\mu x(t) + \sum_{i=1}^{m} p_i\, \mathrm{e}^{-r_i x(t-\tau_i)}, \quad t \geqslant 0, \tag{3}$$

where μ and p_i, r_i, τ_i $(i = 1, 2, \ldots, m)$ are positive constants,

$$x'(t) = -\alpha(t)x(t) + \beta(t)\mathrm{e}^{-x(t-m\omega)}, \quad t \geqslant 0, \tag{4}$$

where α and β are positive ω-periodic functions, and

$$x'(t) = -\alpha(t)x(t) + \beta(t)\mathrm{e}^{-x(t-\tau(t))}, \quad t \geqslant 0, \tag{5}$$

where α, β, τ are positive ω-periodic functions, see Xu and Li [19], Graef et al. [3], Jiang and Wei [7]. In addition, Gopalsamy and Trofimchuk[2] have investigated the existence of a globally attractive almost periodic solution of a single species model given by the nonautonomous Lasota–Wazewska-type delay differential equations

$$x'(t) = -\delta(t)x(t) + p(t)f(x(t-\tau)), \tag{6}$$

where $\tau > 0$, $p(t) \geqslant 0$, $\delta(t)$ are continuous almost periodic functions, and f is a decreasing positive C^1-function.

In Eq. (1), we shall use the following hypotheses:

(H_1) a, p_i and q_i $(i = 1, 2, \ldots, m)$ are positive continuous ω-periodic functions;
(H_2) τ_i $(i = 1, 2, \ldots, m)$ are continuous ω-periodic functions.

For convenience, we shall introduce the notations:

$$\bar{h} = \max_{t\in[0,\omega]} \{h(t)\}, \quad \underline{h} = \min_{t\in[0,\omega]} \{h(t)\},$$