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Nonlinear Analysis 68 (2008) 2943–2956

Nonlinear Analysis

www.elsevier.com/locate/na

Local existence for nonlinear Volterra integrodifferential equations with infinite delay

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Received 20 September 2006; accepted 23 February 2007

Abstract

In this paper, we investigate the local existence and uniqueness of solutions to integrodifferential equations with infinite delay, which are more general than those in previous studies. We assume that the linear part of the equation is nondensely defined and satisfies a Hille–Yosida condition. Moreover, the continuity of solutions with respect to initial conditions is also studied. In order to illustrate our abstract results, we conclude this work with an example.

Keywords: Integrodifferential equation; Infinite delay; Phase space; Hille–Yosida operator

1. Introduction

Integrodifferential equations with delay are important for investigating some problems raised from natural phenomena. They have been studied in many different aspects. The purpose of this paper is to study the local existence and the well posedness of the following integrodifferential differential equation with infinite delay in a Banach space $(X, \|\cdot\|)$

$$\begin{cases} u'(t) = Au(t) + \displaystyle\int_0^t k(t, \theta, u(\theta))\mathrm{d}\theta + F(t, u_t), \quad t \in [0, T], \\ u_0 = \varphi \in \mathcal{P} \end{cases} \tag{1.1}$$

where $A : D(A) \subset X \to X$ is a Hille–Yosida operator with type (M, ω), that is there exist $\omega \in \mathbb{R}$ and $M \geq 1$ such that $(\omega, +\infty) \subset \rho(A)$ ($\rho(A)$ is the usual resolvent set of A) and satisfies

$$\|(\lambda - A)^{-n}\| \leq \frac{M}{(\lambda - \omega)^n} \quad \text{for all } \lambda > \omega \text{ and } n \in \mathbb{N}.$$

The domain $D(A)$ may be nondense in X. Moreover, $D(A)$ endowed with the graph norm $\|\cdot\|_{D(A)}$ becomes a Banach space. The map $x \mapsto k(t, s, x)$ is assumed to be defined from $(D(A), \|\cdot\|_{D(A)})$ to X. Furthermore, F is an X-valued

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0362-546X/$ - see front matter
doi:10.1016/j.na.2007.02.038

function defined on $[0, T] \times \mathcal{P}$ where $\mathcal{P}$ is a linear space of functions mapping $(-\infty, 0]$ into X satisfying some axioms which will be described later, and the function $u_t(\cdot) \in \mathcal{P}$ is defined by

$$u_t(\theta) = u(t + \theta) \quad \text{for } \theta \in (-\infty, 0].$$

As in [11], we consider a nonlinear Volterra integrodifferential equation of hyperbolic type

$$\begin{cases} u_t(t, x) = Au(t, x) + \int_0^t g(t, s, u(s, x), \ldots, D_x^\beta u(s, x))\mathrm{d}s + f(t, x), & (t, x) \in [0, \infty) \times \Omega, \\ Bu(t, x) = 0, \quad (t, x) \in [0, \infty) \times \partial\Omega, \\ u(0, x) = z(x), \quad x \in \Omega, \end{cases} \tag{1.2}$$

where A and B denote suitable linear partial differential operators on a subset Ω of $\mathbb{R}^n$ and on its boundary $\partial\Omega$, respectively, and the kernel g does not depend on the derivatives $D_x^\beta u$ of order greater than the order of A. The abstract version of the initial boundary value problem (1.2) is given by

$$\begin{cases} u'(t) = Au(t) + \int_0^t k(t, \theta, u(\theta))\mathrm{d}\theta + f(t), \quad t \in [0, T], \\ u(0) = z \in D(A). \end{cases} \tag{1.3}$$

Existence and uniqueness were proved for Eq. (1.3) under some suitable assumptions. Some properties of the solution were also studied. A vast literature has investigated this equation in various aspects. For the study of Eq. (1.3), we also refer the reader to [6,7,12] and [16].

Equations with delay appear in many mathematical models of natural phenomena. Recently, the following differential equations with delay have been studied by many authors ([1,2,5,8–10,13–15], and references therein):

$$\begin{cases} u'(t) = Au(t) + F(t, u_t), \quad t \in [0, T], \\ u_0 = \varphi \in \mathcal{P}. \end{cases} \tag{1.4}$$

There has been a great deal of work contributed to the study of partial differential equations with delay by using different methods under different conditions. The most classical work is due to Travis and Webb [15]. The investigation of functional differentials with infinite delay in an abstract admissible phase space was initiated by Hale and Kato [5], Kappel and Schappacher [9], and Schumacher [14]. The method of using admissible phase spaces enables one to treat a large class of functional differential equations with infinite delay at the same time and obtain general results. For a detailed discussion on this topic, we refer the reader to the book by Hino et al. [8].

Eq. (1.1) is the mixed type of Eqs. (1.3) and (1.4). It will enable us to study the nonlinear Volterra integrodifferential equation with delay. On the basis of the results in Eq. (1.4), we generalize the method used in [11] to derive local existence and uniqueness of Eq. (1.1).

In Section 2, we recall some preliminary results about Eqs. (1.3) and (1.4). Some basic notation and assumptions are also given in this section. In Section 3, we prove the local existence and uniqueness of solutions to Eq. (1.1) which are the main results of this paper. Moreover, some properties of solutions are also studied. In Section 4, we give an example to show that our results are valuable.

2. Preliminaries

Through out this paper A denotes a Hille–Yosida operator with type (M, ω), $\Delta(0, T)$ denotes the set $\{(t, s); 0 \leq s \leq t \leq T\} \subseteq \mathbb{R}^2$ where T is a positive number, and D is the Banach space $D(A)$ equipped with graph norm $\|\cdot\|_{D(A)}$. Consider the function $k \in C(\Delta(0, T) \times D, X)$. We make the following assumptions.

(H1) The derivative $k_t(t, s, x)$ exists and is continuous from $\Delta(0, T) \times D$ into X. k and k_t satisfy the Lipschitz condition, i.e. there exists B such that

$$\|k(t, s, x) - k(t, s, y)\| \leq B\|x - y\|_{D(A)}$$

and

$$\|k_t(t, s, x) - k_t(t, s, y)\| \leq B\|x - y\|_{D(A)}$$

for each $(t, s) \in \Delta(0, T)$ and $x, y \in D(A)$.