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Existence and regularity in the α-norm for some neutral partial differential equations with nonlocal conditions

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Abstract

In this work, we discuss the existence and regularity of solutions for some partial differential equations with nonlocal conditions in the α-norm. We assume that the linear part generates an analytic semigroup and the nonlinear part is a Lipschitz continuous function with respect to the fractional power norm of the linear part.
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1. Introduction

In this work, we study the existence and regularity of solutions in the α-norm for partial differential equations with nonlocal condition. The following model provides an example of such a situation

$$\begin{cases} \dfrac{\partial}{\partial t}\left[u(t,\xi) - \displaystyle\int_0^\pi b(t,\xi,\sigma)u(\sin t,\sigma)\mathrm{d}\sigma\right] = \dfrac{\partial^2}{\partial \xi^2}\left[u(t,\xi) - \displaystyle\int_0^\pi b(t,\xi,\sigma)u(\sin t,\sigma)\mathrm{d}\sigma\right] \\ \qquad\qquad + \chi\left(t, \dfrac{\partial}{\partial \xi}u(\sin t,\xi)\right), \quad 0 \le t \le 1 \text{ and } 0 \le \xi \le \pi, \\ u(t,0) = u(t,\pi) = 0, \quad 0 \le t \le 1, \\ u(0,\xi) + \displaystyle\int_0^\pi k(\xi,\sigma)u(t,\sigma)\mathrm{d}\sigma = z_0(\xi), \quad 0 \le \xi \le \pi, \end{cases} \tag{1.1}$$

where b, χ and k are given functions. Eq. (1.2) can be written as a neutral partial differential equation of the following form

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t}[x(t) - F(t, x(h_1(t)))] = -A[x(t) - F(t, x(h_1(t)))] + G(t, x(h_2(t))), \quad 0 \le t \le a, \\ x(0) + g(x) = x_0 \in X, \end{cases} \tag{1.2}$$

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where $-A$ generates an analytic semigroup $(T(t))_{t\geq 0}$ on a Banach space X. The functions F, G, g, h_1 and h_2 are continuous functions and will be specified later. Recall that the nonlocal condition $x(0) + g(x) = x_0$ is much better than the classical condition $x(0) = x_0$ and it has many applications in physical systems and were the subject of many works, for more details, we refer to [1–4,6,7,9,10]. In [3], the authors, investigated the existence of mild solutions of Eq. (1.2) with $F(.,.) = 0$. The existence of strong solutions is considered only when X is reflexive. More precisely, the authors proved that any mild solution $x(.)$ satisfying the following condition

$$\|x(h_2(t_2)) - x(h_2(t_1))\| \leq L\|x(t_2) - x(t_1)\| \tag{1.3}$$

is a strong solution. However, condition (1.3) is very difficult to be satisfied, since mild solutions are usually unknown. In [7], the authors studied the existence of mild solutions and strong solutions in a reflexive Banach space of the following neutral partial differential equation

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t}[x(t) + F(t, x(h_1(t)))] = -Ax(t) + G(t, x(h_2(t))), \quad 0 \leq t \leq a, \\ x(0) + g(x) = x_0 \in X. \end{cases} \tag{1.4}$$

In [8,5], the authors established several results on the existence and regularity of solutions for Eq. (1.2) when $F(,) = 0$ and A is not necessarily densely defined. Recently in [6], the authors, studied the existence and regularity of the mild solution when G is defined in the whole space X and Lipschitz continuous with the X-norm. Here we suppose that G is defined in smaller space than X, namely, $D(A^\alpha)$, for some $0 < \alpha < 1$, the domain of the fractional power of A. We discuss in general Banach spaces, the existence and regularity of solutions for Eq. (1.2) with α-norm. We prove the existence of strict solutions which are much better than strong solutions. This work is organized as follows, in Section 2, we study the existence of mild and strict solutions. For illustration in the last section, we propose to study the existence of solutions for the model (1.1).

2. Main results

In this work, we assume that

($\mathbf{H}_0$) $-A$ is the infinitesimal generator of an analytic semigroup $(T(t))_{t\geq 0}$ on a Banach space X and $0 \in \rho(A)$, where $\rho(A)$ is the resolvent set of A.

Then, there exist constants $M_0 \geq 1$ and $\omega \in \mathbb{R}$ such that $|T(t)| \leq M_0 e^{\omega t}$ for $t \geq 0$. Without loss of generality, we assume that $\omega \geq 0$. If the assumption $0 \in \rho(A)$ is not satisfied, one can substitute the operator A by the operator $(A - \sigma I)$ with σ large enough such that $0 \in \rho(A - \sigma I)$ and so we can always assume that $0 \in \rho(A)$.

For the fractional power $(A^\alpha, D(A^\alpha))$, for $0 < \alpha < 1$, and its inverse $A^{-\alpha}$, one has the following known result.

Theorem 2.1 *([11], pp. 69–75). Let $0 < \alpha < 1$ and assume that* ($\mathbf{H}_0$) *holds. Then,*

(i) *$D(A^\alpha)$ is a Banach space with the norm $|x|_\alpha = |A^\alpha x|$ for $x \in D(A^\alpha)$,*
(ii) *$T(t) : X \to D(A^\alpha)$ for $t > 0$,*
(iii) *$A^\alpha T(t)x = T(t)A^\alpha x$ for $x \in D(A^\alpha)$ and $t \geq 0$,*
(iv) *for every $t > 0$, $A^\alpha T(t)$ is bounded on X and there exists $M_\alpha > 0$ such that*

$$|A^\alpha T(t)| \leq M_\alpha \frac{e^{\omega t}}{t^\alpha} \quad \text{for } t > 0, \tag{2.1}$$

(v) *$A^{-\alpha}$ is a bounded linear operator in X with $D(A^\alpha) = \mathrm{Im}\,(A^{-\alpha})$,*
(vi) *if $0 < \alpha < \beta \leq 1$, then $D(A^\beta) \hookrightarrow D(A^\alpha)$.*

Let $X_\alpha = D(A^\alpha)$.

Definition 2.2. A continuous function $x(.) : [0, a] \to X_\alpha$ is said to be a mild solution of Eq. (1.2), if

(i) $x(t) = T(t)[x(0) - F(0, x(h_1(0)))] + F(t, x(h_1(t))) + \int_0^t T(t-s)G(s, x(h_2(s)))\mathrm{d}s, 0 \leq t \leq a.$
(ii) $x(0) + g(x) = x_0$.

Definition 2.3. A continuous function $x(.) : [0, a] \to X_\alpha$ is said to be a strict solution of Eq. (1.2), if

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