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Existence of three positive solutions for m-point boundary-value problems with one-dimensional p-Laplacian[☆]

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Abstract

In this paper, we consider the multipoint boundary value problem for the one-dimensional p-Laplacian

$$(\phi_p(u'))' + q(t)f(t, u(t), u'(t)) = 0, \quad t \in (0, 1),$$

subject to the boundary conditions:

$$u(0) = 0, \qquad u(1) = \sum_{i=1}^{m-2} a_i u(\xi_i),$$

where $\phi_p(s) = |s|^{p-2}s$, $p > 1$, $\xi_i \in (0, 1)$ with $0 < \xi_1 < \xi_2 < \cdots < \xi_{m-2} < 1$ and $a_i \in [0, 1)$, $0 \le \sum_{i=1}^{m-2} a_i < 1$. Using a fixed point theorem due to Avery and Peterson, we study the existence of at least three positive solutions to the above boundary value problem. The interesting point is that the nonlinear term f explicitly involves a first-order derivative.

Keywords: Multipoint boundary value problem; Avery–Peterson's fixed point theorem; Positive solution; One-dimensional p-Laplacian

1. Introduction

In this paper, we study the existence of multiple positive solutions to the boundary value problem (BVP for short) for the one-dimensional p-Laplacian

$$(\phi_p(u'))' + q(t)f(t, u(t), u'(t)) = 0, \quad t \in (0, 1), \tag{1.1}$$

$$u(0) = 0, \qquad u(1) = \sum_{i=1}^{m-2} a_i u(\xi_i), \tag{1.2}$$

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where $\phi_p(s) = |s|^{p-2}s$, $p > 1$, $\xi_i \in (0, 1)$ with $0 < \xi_1 < \xi_2 < \cdots < \xi_{m-2} < 1$ and a_i, f satisfy

(H_1) $a_i \in [0, 1)$ satisfy $0 \le \sum_{i=1}^{m-2} a_i < 1$;
(H_2) $f \in C([0, 1] \times [0, +\infty) \times R, (0, +\infty))$;
(H_3) $q(t) \in L^1[0, 1]$ is nonnegative on $(0, 1)$ and $q(t)$ is not identically zero on any subinterval of $(0, 1)$. Furthermore, $q(t)$ satisfies $0 < \int_0^1 q(t)\mathrm{d}t < \infty$.

The study of multipoint boundary value problems for linear second-order ordinary differential equations was initiated by Il'in and Moiseev [1]. Since then there has been much current attention focused on the study of nonlinear multipoint boundary value problems, see [2–7].

For the problem (1.1), when the nonlinear term f does not depend on the first-order derivative, many authors studied the equation

$$(\phi_p(u'))' + q(t)f(t, u) = 0, \quad t \in (0, 1), \tag{1.3}$$

with different boundary conditions. For example, in [5], Ma and Ge studied the existence of positive solutions for the multipoint BVP,

$$\begin{aligned} &(\phi_p(u'))' + q(t)f(t, u) = 0, \quad t \in (0, 1),\\ &u'(0) = \sum_{i=1}^{m-2} \alpha_i u'(\xi_i), \qquad u(1) = \sum_{i=1}^{m-2} \beta_i u(\xi_i). \end{aligned}$$

The main tool is the monotone iterative technique.

In [6,7], Wang and Ge considered multipoint BVPs for the one-dimensional p-Laplacian successively

$$\begin{aligned} &(\phi_p(u'))' + f(t, u) = 0, \quad t \in (0, 1),\\ &\phi_p(u'(0)) = \sum_{i=1}^{n-2} a_i \phi_p(u'(\xi_i)), \qquad u(1) = \sum_{i=1}^{n-2} a_i u(\xi_i) \end{aligned}$$

and

$$\begin{aligned} &(\phi_p(u'))' + f(t, u) = 0, \quad t \in (0, 1),\\ &u'(0) = \sum_{i=1}^{n-2} \alpha_i u'(\xi_i), \qquad u(1) = \sum_{i=1}^{n-2} \beta_i u(\xi_i). \end{aligned}$$

They provided sufficient conditions for the existence of multiple positive solutions to the above BVPs by applying a fixed point theorem in a cone.

However, all the above works about positive solutions were done under the assumption that the first order derivative is not involved explicitly in the nonlinear term. Recently, Bai, Gui and Ge in [8], investigated the following two-point BVPs

$$\begin{aligned} &(\phi_p(u'))' + q(t)f(t, u(t), u'(t)) = 0, \quad t \in (0, 1),\\ &\alpha\phi_p(u(0)) - \beta\phi_p(u'(0)) = 0, \qquad \gamma\phi_p(u(1)) + \delta\phi_p(u'(1)) = 0 \end{aligned}$$

or

$$u(0) - g_1(u'(0)) = 0, \qquad u(1) + g_2(u'(1)) = 0,$$

where $g_1(t)$ and $g_2(t)$ are continuous functions defined on $(-\infty, +\infty)$, $\alpha > 0$, $\beta \ge 0$, $\gamma > 0$, $\delta \ge 0$.

Wang and Ge in [9] considered the following two-point BVPs

$$\begin{aligned} &(\phi_p(u'))' + q(t)f(t, u(t), u'(t)) = 0, \quad t \in (0, 1),\\ &u(0) = 0, \qquad u(1) = 0, \end{aligned}$$

or

$$u(0) = 0, \qquad u'(1) = 0.$$