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Periodic solutions for a class of higher-dimension functional differential equations with impulses[☆]

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Abstract

By employing a fixed-point theorem in cones, we establish some criteria for existence of positive periodic solutions of a class of n-dimension periodic functional differential equations with impulses. We also give some applications to several biomathematical models and new results are obtained.

MSC: 39A10; 39A12

Keywords: Functional differential equations; Impulse; Positive periodic solution; Fixed-point theorem

1. Introduction

Some evolution processes are distinguished by the circumstance that the evolutions change very rapidly at certain instants. In mathematical simulations, the sudden change can be described by the impulsive differential equations. As for the differential equations with impulses, some properties have been studied such as oscillation, asymptotic behavior, stability and existence of solutions by many authors [1–4]. Recently, by employing the powerful and efficient method of coincidence degree [5–7] and theory in cones [6,8], some results on periodic solutions have been obtained. However, only a little work has been done on the existence of positive periodic solutions to the high-dimension impulsive differential equations based on the theory in cones. Motivated by this, in this paper, we mainly consider the n-dimension differential equation with impulses as follows:

$$\begin{aligned} &\dot{x}(t) = A(t)x(t) + f(t, x_t), \quad t \neq \tau_k, k \in Z_+, \\ &\triangle x|_{t=\tau_k} = I_k(x(\tau_k)), \end{aligned} \tag{1.1}$$

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where $A(t) = \mathrm{diag}[a_1(t), a_2(t), \ldots, a_n(t)]$, $a_j \in C(R, R_+)$ is ω-periodic with $\overline{a}_j = \frac{1}{\omega}\int_0^\omega a_j(t)\mathrm{d}t > 0$ $(j = 1, 2, \ldots, n)$; $f(t, x_t)$ is an operator defined on $R \times BC(R, R^n)$ (here $BC(R, R^n)$ denotes the Banach space of bounded continuous operator $\phi : R \to R^n$ with the norm $\|\phi\| = \sum_{i=1}^n \sup_{\theta \in R} |\phi_i(\theta)|$, where $\phi = (\phi_1, \phi_2, \ldots, \phi_n)^{\mathrm{T}}$, $f(t + \omega, x_t) = f(t, x_t)$ and $\triangle x|_{t=\tau_k} = x(\tau_k^+) - x(\tau_k)$ (here $x(\tau_k^+)$ represents the right limit of $x(t)$ at the point τ_k); $I_k = (I_k^1, I_k^2, \ldots, I_k^n) \in C(R_+^n, R_-^n)$, i.e., x changes decreasingly suddenly at times τ_k; $\omega > 0$ is a constant, Z_+, R_+ and R_- are the sets of all positive integers, and all nonnegative and nonpositive real numbers, respectively. We assume that there exists an integer $p > 0$ such that $\tau_{k+p} = \tau_k + \omega$, $I_{k+p} = I_k$, where $0 < \tau_1 < \tau_2 < \cdots < \tau_p < \omega$.

It is well known that the functional differential system (1.1) includes many mathematical ecological models. For example, the delayed periodic single-species logistic model [9,10] with impulses

$$\begin{aligned} &\dot{x}(t) = x(t)\left[a(t) - \sum_{i=1}^n b_i(t)x(t - \tau_i(t))\right], \quad t \neq t_k,\ k \in Z_+, \\ &\triangle x|_{t=t_k} = I_k(x(t_k)); \end{aligned} \tag{1.2}$$

the Michaelis–Menton single-species growth model [10] with impulses

$$\begin{aligned} &\dot{x}(t) = a(t)x(t)\left[1 - \sum_{i=1}^n \frac{b_i(t)x(t - \tau_i(t))}{1 + c_i(t)x(t - \tau_i(t))}\right], \quad t \neq t_k,\ k \in Z_+, \\ &\triangle x|_{t=t_k} = I_k(x(t_k)); \end{aligned} \tag{1.3}$$

and the celebrated n-species Lotka–Volterra competition system with distributed infinity delay and impulses

$$\begin{aligned} &\dot{x}_i(t) = x_i(t)\left[a_i(t) - \sum_{j=1}^n \int_{-\infty}^0 x_j(t + s)\mathrm{d}\mu_{ij}(t, s)\right], \quad t \neq t_k,\ k \in Z_+, \\ &\triangle x_i|_{t=t_k} = I_{ik}(x(t_k)), \quad i = 1, 2, \ldots, n, \end{aligned} \tag{1.4}$$

where $a, a_i, b_i, c_i, \tau_i, r_i \in C(R, R_+)(i = 1, 2, \ldots, n)$ are all ω-periodic functions; $\mu_{ij}(t, s)$ is a continuous ω-periodic function with respect to t, and is nondecreasing bounded and continuous from the left in s, with $\int_{-\infty}^0 \mathrm{d}\mu_{ij}(t, s) < \infty$, $i, j = 1, 2, \ldots, n$; and there exists $p \in Z_+$ such that $t_{k+p} = t_k + \omega$ and $I_{k+p} = I_k$, where $0 < t_1 < t_2 < \cdots < t_p < \omega$.

The main purpose of this paper is to obtain the existence of positive periodic solutions of (1.1) and the main results based on theory in cones will be given in Section 2. In the last section, we will apply the results established to the above impulsive delayed population models (1.2)–(1.4); some new results are obtained.

Throughout of this paper, we shall use the following notation. Let $J \subset R$; denote by $PC(J, R^n)$ the set of operators $\phi : J \to R^n$ which are continuous for $t \in J, t \neq \tau_k$ and have discontinuities of the first kind at the points $\tau_k \in J(k \in Z_+)$, but are continuous from the left at these points. For each $x = (x_1, x_2, \ldots, x_n)^{\mathrm{T}} \in R^n$, the norm of x is defined as $|x| = \sum_{i=1}^n |x_i|$. The matrix $A > B$ $(A \leq B)$ means that each pair of corresponding elements of A and B satisfies the inequality "$>$" ("$\leq$"). In particular, A is called a positive matrix if $A > 0$.

2. Main results

In what follows,we always assume that

(H_1) $f(t, \varphi_t) \leq 0$ for all $(t, \varphi) \in R \times BC(R, R_+^n)$;
(H_2) $f_i(t, \varphi_t)$ is a continuous function of t for each $\varphi \in BC(R, R_+^n)$, $i = 1, 2, \ldots, n$;
(H_3) for any $L > 0$ and $\varepsilon > 0$, there exists $\delta > 0$ such that for $\phi, \psi \in BC(R, R_+^n)$, $|\phi| \leq L$, $\|\psi\| \leq L$ and $\|\phi - \psi\| \leq \delta$ imply that

$$|f_i(t, \phi_t) - f_i(t, \psi_t)| < \varepsilon, \quad \forall t \in [0, \omega],\ i = 1, 2, \ldots, n.$$

For convenience, we shall give some definitions on cones firstly.

Definition (*[11]*). Let X be a Banach space and P be a closed, nonempty subset of X. P is a cone if