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Necessary and sufficient conditions for stabilization of discrete-time planar switched systems

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Abstract

This paper discusses the stabilization problem for single-input discrete-time planar switched systems. We establish necessary and sufficient conditions for the stabilization of planar switched systems. A series of linear inequalities are presented for describing the set of all common quadratic Lyapunov functions. Our results are not only easily testable, but also constructive.

Keywords: Switched systems; Stabilization; Common quadratic Lyapunov function; Brunovsky canonical form; Necessary and sufficient conditions

1. Introduction

Switched systems are an important class of hybrid systems which consist of a family of subsystems and a switching rule specifying which subsystem will be activated along the system trajectory at each instant of time. In the last decade, switched systems have been investigated by many authors (see [2,4,8,15] and the references therein). This problem is not only theoretically interesting but also practically important. Many real-world systems such as chemical processes, transportation systems, computer controlled systems, power systems and communication networks can be modeled as switched systems (see [5,6,14,16–19]).

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One way to solve the stabilization problem is to find a common quadratic Lyapunov function. Finding a common quadratic Lyapunov function for a family of subsystems is still an open problem, even though some progress has been made [1,7,9–13]. Recently, Cheng [3] studied the single-input planar continuous-time switched control system

$$\dot{x}(t) = A_{\sigma(x,t)}x(t) + B_{\sigma(x,t)}u(t), \tag{1}$$

where $\sigma(x,t)\colon R^2 \times [0,\infty) \to \mathcal{N} = \{1,2,\ldots,N\}$ is an arbitrary mapping. The author established a necessary and sufficient condition for system (1) to be stabilizable. To the best of our knowledge, there is no corresponding condition in the literature for the discrete-time planar switched control system

$$x_{k+1} = A_{\sigma(x,k)}x_k + B_{\sigma(x,k)}u_k, \tag{2}$$

where $\sigma(x,k)\colon R^2 \times \{0,1,2,\ldots,\} \to \mathcal{N}$ is an arbitrary mapping.

Extension from continuous-time switched systems to discrete-time switched systems is far from trivial. In fact, discrete-time switched systems are much more difficult to deal with than continuous-time ones. This is due to the fact that in continuous time, only the boundary of the level sets to the Lyapunov equation needs to be considered. In the following section, we will study the stabilization problem for the discrete-time planar switched system (2). We shall establish necessary and sufficient conditions for the existence of common quadratic Lyapunov functions. A series of linear inequalities will be presented for describing the set of all common quadratic Lyapunov functions.

The paper is organized as follows. Section 2 considers the stabilization of single-input discrete-time planar switched systems. Two necessary and sufficient conditions are established. An algorithm for solving the stabilization problem is given in Section 3. Section 4 presents two numerical examples illustrating the effectiveness of our theoretical results and algorithm. Section 5 contains the conclusion.

Notations. We use standard notation throughout this paper. M^{T} is the transpose of the matrix M. $M > 0$ ($M < 0$) means that M is positive definite (negative definite).

2. Stabilization of discrete-time planar switched systems

First, we give several definitions to clearly formulate the problem to be discussed in this section.

Definition 1. The discrete-time switched system $x_{k+1} = A_{\sigma(x,k)}x_k$ is said to share a common quadratic Lyapunov function, $x^{\mathrm{T}}Px$ with $P > 0$, if

$$A_i^{\mathrm{T}}PA_i - P < 0, \qquad i \in \mathcal{N}.$$

Definition 2. System (2) is said to be stabilizable with observable $\sigma(x,k)$ if there exist a matrix $P > 0$ and a set of state feedback controls $u = K_i x$ such that the closed-loop system of (2) shares a common quadratic Lyapunov function, $x^{\mathrm{T}}Px$.

It is easy to see that a necessary condition for system (2) to be stabilizable is that each subsystem is stabilizable. So a reasonable assumption is

A1. All the subsystems are stabilizable.

The following lemma shows that, without loss of generality, we can replace A1 by

A2. All the subsystems are reachable.