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On conditions for regularity of solutions for a piecewise linear system

Jan Melin[a,*], Anders Hultgren[b]

[a] *Department of Chemistry and Biomedical Sciences, University of Kalmar, 391 82 Kalmar, Sweden*
[b] *Department of Technology, University of Kalmar, S-391-29 Kalmar, Sweden*

Abstract

We study a piecewise linear system approximating the behaviour of a switched DC/DC converter. We give conditions for a unique limit cycle in our different cases and give examples of complicated phenomena that might occur. The stability of a state space trajectory controlled converter is addressed.

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Keywords: Resonant converters; Piecewise linear system; Limit cycles; Switched systems

1. Introduction

A group of physical systems can be characterized by abrupt changes in dynamics. Modelling that group of systems as a switched system has gained increased attention during the last few years. See for instance [12–14]. The switched model enables more accurate analysis and more advanced control of the physical system compared to modelling with averaging methods or ignoring part of the dynamics. The resulting control algorithms are often calculation intensive but the development in real time capacity makes them possible to implement.

General analysis and control of switched systems is difficult and this report focuses on the principal behaviour of a certain controlled switched system, a switched electrical power converter. Applying switched control of a switched system, like an electrical power converter, can give rise to mathematically interesting solutions of the switched differential equations describing

* Corresponding author. Tel.: +46 480 446149, +46 480 446304; fax: +46 480 446099, +46 480 446330.
E-mail addresses: jan.melin@hik.se (J. Melin), anders.hultgren@hik.se (A. Hultgren).

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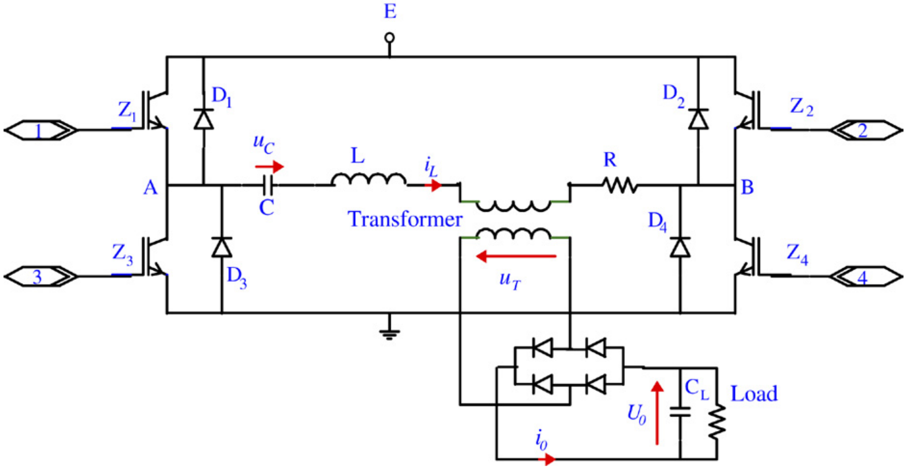

Fig. 1.1.

the controlled converter. Global analysis of the switched system is of course crucial when designing switched control of the physical system. It is of utmost importance to avoid possible chattering phenomena.

The use of switched electrical power converters is widespread due to the extensive use of electronic equipment. The demand for high bandwidth performance motivates modelling and control of switched converters as switched systems. The analysed converter is of resonant type, having many advantages such as low switching losses at higher switching frequencies (zero-current switching, ZCS, or zero-voltage switching, ZVS), and easier electromagnetic interference (EMI) filtering. However the control will be more complex compared to pulse width modulation, PWM, controlled converters, [11]. The output of the converters is the current on the secondary side of a transformer, $i_0(t)$; see Fig. 1.1. The resonant converter is applied in high voltage equipment with a capacitive load property. The capacitive load and the high transformer ratio, n, will give a comparatively large load capacitance, making the load voltage, v_0, only slowly varying. The converter is controlled by the supply voltage, E, and the four transistors Z_1–Z_4 determining the voltage across the resonant circuit, U_{AB}, between junctions A and B. The control of the converter is performed with non-linear state space feedback, determining the voltage U_{AB} to be E, $-E$ or 0.

Different control methods have been proposed for the control of series resonant converters. A rather general non-linear control method is chosen in this report implying state region feedback similar to that for the controllers suggested in [10], and later by many authors, e.g. [26]. The closed system can in this case be characterized as a switched system; in fact it is piecewise linear. Recent works in modelling and analysis of switched converters as a switched system can be found in [15,18]. Analysis of piecewise linear systems can be found in [17,16,14] for instance.

This report is devoted to a mathematical analysis of the rich kinds of solutions of the switched differential equations that can occur in a controlled switched system of the presented type.

Our switched model is a planar, linear system with discontinuous right hand side, also called a Filippov system [1]. We have two discontinuity curves in the model, one a circle centred at the origin and the other the x_1-axis [5]. We will define and prove regularity in most parts of