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Existence and asymptotic behavior of non-radially symmetric ground states of semilinear singular elliptic equations[☆]

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Abstract

This paper deals with the existence and asymptotic behavior of entire positive solutions of the semilinear elliptic equation $-\Delta u = \rho(x) f(u)$ in $\mathbf{R}^N$ $(N \geq 3)$, where ρ is nonnegative and locally Hölder continuous and f is a positive locally Lipschitz continuous function, singular at 0 in the sense that $f(r) \overset{r\to 0}{\longrightarrow} \infty$. No symmetry is required from ρ and no monotonicity condition is imposed on f. Arguments for lower and upper solutions are exploited.

Keywords: Semilinear elliptic equations; Ground states; Asymptotic behavior; Lower–upper solutions

1. Introduction

In this work we study the existence and asymptotic behavior of entire solutions of the problem

$$\begin{cases} -\Delta u = \rho(x) f(u) & \text{in } \mathbf{R}^N, \\ u > 0, \quad u(x) \overset{|x|\to\infty}{\longrightarrow} 0, \end{cases} \tag{1.1}$$

where $N \geq 3$, $\rho : \mathbf{R}^N \to [0, +\infty)$ is continuous and $f : (0, +\infty) \to (0, +\infty)$ belongs to $Lip_{loc}((0, \infty))$ with f singular at 0 in the sense that $f(r) \overset{r\to 0}{\longrightarrow} \infty$. No symmetry is required for

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ρ and no monotonicity condition is imposed on f. This sort of problem appears in the physical sciences and has been exhaustively investigated in the last few years.

As regards corresponding problems on bounded domains, we refer the reader to the pioneering work [5] by Crandall, Rabinowitz and Tartar, where both perturbation and global bifurcation methods are exploited and even linear operators more general than Δ are considered. We also refer the reader to the work of Lazer and McKenna [13], where singular problems involving the Monge–Ampére operator rather than Δ are investigated.

Returning to problems on the whole space, as a brief historical account of problems closer to our interests in the present paper, we recall that Edelson in [9] and Shaker in [14] addressed the cases $f(r) = r^{-\lambda}$ with $\lambda \in (0, 1)$ and $\lambda \in (0, \infty)$ respectively, and additionally assumed

$$\int_1^\infty s^{N-1+\lambda(N-2)} \varphi(r) \mathrm{d}r < \infty, \tag{1.2}$$

where $\varphi(r) := \max_{|x|=r} \rho(x)$ while Lair and Shaker in [12] assumed $\lambda \in (0, \infty)$ and

$$\int_1^\infty r\varphi(r)\mathrm{d}r < \infty. \tag{1.3}$$

Zhang in [16] improved on the results above by exploiting C^1-nonlinearities f, singular at 0, satisfying the condition $f' < 0$ which includes pure powers as a special case. Cirstea and Radulescu in [4] showed the existence of entire solutions of (1.1) in the case of a C^1-term f, singular at 0, satisfying the following additional conditions:

$$\lim_{r\to 0} \frac{f(r)}{r} = \infty,\ f \text{ bounded at } \infty,\ \frac{f(r)}{r+b} \text{ decreasing for some } b > 0.$$

There is now a broad literature on singular problems. We refer the reader to the recent work [8] by Dinu, where the condition "$f(r)/r$ is decreasing in $(0, \infty)$" which is weaker than the related one in [4] mentioned above is applied. We also refer the reader to Cirstea, Ghergu and Radulescu [3], Ghergu and Radulescu [10], Dinu [7] and their references.

The main result of this paper is the following.

Theorem 1.1. *Assume $\rho \in C^{0,\alpha}_{loc}$ for some $\alpha \in (0, 1)$. Assume also (1.3) and*

$$\text{(i) } \frac{f(r)}{r} \textit{ is decreasing},\ \text{(ii) } \lim_{r\to 0} \frac{f(r)}{r} = \infty,\ \text{(iii) } \lim_{r\to +\infty} \frac{f(r)}{r} = 0. \tag{1.4}$$

Then (1.1) admits a solution $u \in C^{2,\alpha}_{loc}$. If, in addition,

$$\lim_{|x|\to\infty} |x|^\mu \varphi(|x|) < \infty, \tag{1.5}$$

for some $\mu \in (2, N)$, then

$$u(x)^2[f(u(x)/4)]^{-1} = O(|x|^{(2-\mu)}) \textit{ as } |x| \to \infty. \tag{1.6}$$

Remark 1. (i) The class of functions

$$f(r) = r^{-\lambda} + r^\gamma, \quad r > 0,$$

where $\lambda > 0$ and $0 \le \gamma \le 1$ provides examples of singular nonlinearities, covered by Theorem 1.1, but not by the results referred to above. To the best of our knowledge a decay rate such as (1.6) was not shown earlier in the study of (1.1).