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An inverse heat conduction problem with a nonlinear source term

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Abstract

In this paper, the nonlinear problem of an inhomogeneous heat equation with linear boundary conditions will be considered. The surface heat flux history of a heated conducting body will be identified. The approach of the proposed method is to approximate the unknown function using linear polynomial pieces which are determined consecutively from the solution of the minimization problem on the basis of overspecified data. Some numerical examples will also be presented.

MSC: 35R30

Keywords: Direct and inverse heat conduction problem; Residual minimization; Chebyshev polynomial; Least-squares method; Implicit finite difference approximation

1. Introduction

Determination of the unknown source term in a parabolic differential equation has been discussed by several authors [3,4,7–9]. Cannon and DuChateau considered the identification of an unknown state-dependent source term in a reaction–diffusion equation [5]. In all of these works, the initial conditions and boundary conditions are considered as known functions and the identification of the unknown source term or thermophysical properties of the body is investigated. In these works, the insufficiency of the input information is compensated by

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there being some additional information on the surface. One of the applications may be in the determination of the surface heat flux histories when re-entering heat shields.

In this paper, we shall deal with the identification of heat flux histories at $x = 0$ in an inverse heat conduction problem, with a nonlinear source term. In fact the basic objective is to find a function from given measurements that are remote in some sense. We consider the following inverse problem:

$$\partial_t u(x,t) - \partial_{xx} u(x,t) = F(u(x,t)), \qquad \text{in } Q_T = \{(x,t) \mid 0 < x < 1, 0 < t < T\}, \tag{1}$$

$$u(x,0) = f(x), \qquad 0 < x < 1, \tag{2}$$

$$\partial_x u(0,t) = q(t), \qquad 0 < t < T, \tag{3}$$

$$\partial_x u(1,t) = g(t), \qquad 0 < t < T, \tag{4}$$

where F, f and g are considered as known functions, while $q(t)$ and $u(x,t)$ are unknown functions. In order to determine $\partial_x u(0,t) = q(t)$, let us consider additional temperature measurements given at the boundary, $x = 1$:

$$u(1,t) = p(t), \qquad 0 \le t \le T. \tag{5}$$

$F(u)$ is a function of the state variable and does not depend explicitly on position or time. In the context of heat conduction or diffusion, the function $F(u)$ is interpreted as a heat or material source, while in a chemical or biochemical application, $F(u)$ may be interpreted as a reaction term.

This paper is organized as follows. In the next section, we will solve the direct heat conduction problem with a nonlinear source term; the process is based on assumptions regarding F, g, f and q possessing a unique solution. In Section 3, we solve the inverse problem based on the implicit finite difference approximation and minimization least-squares method. Some numerical results will be given in Section 4.

2. A direct initial boundary value problem

In this section, let us consider the initial boundary value problem (1)–(4), where F, f, g, and q are known functions and are assumed to satisfy:

(a) $f, g, q \in C[0, \infty)$.
(b) The function $F(u(x,t))$ is a given piecewise differentiable function on the set $\{u \mid -\infty < u < \infty\}$.
(c) There exists a constant C_F such that

$$|F(u) - F(v)| \le C_F |u - v|.$$

(d) F is a bounded and uniformly continuous function in u.

Let the above conditions (a) to (d) be fulfilled; then $u(x,t)$, called the solution of the direct problem (1)–(4), has the form [2]

$$\begin{aligned} u(x,t) = {} & \int_0^1 \{\theta(x-\xi,t) + \theta(x+\xi,t)\} f(\xi)\mathrm{d}\xi - 2\int_0^t \theta(x,t-\tau)q(\tau)\mathrm{d}\tau \\ & + 2\int_0^t \theta(x-1,t-\tau)g(\tau)\mathrm{d}\tau \\ & + \int_0^t \int_0^1 \{\theta(x-\xi,t-\tau) + \theta(x+\xi,t-\tau)\} F(u(\xi,\tau))\mathrm{d}\xi\mathrm{d}\tau, \end{aligned} \tag{6}$$