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An age structured population model in a spatially heterogeneous environment: Existence and uniqueness theory

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Abstract

This paper considers the existence and uniqueness of solutions of an age-structured population model in a spatially heterogeneous bounded environment. The model describes individual organism movement of diffusion and advection in the older life stage but assumes stationarity of individuals in the embryonic stage. The mathematical formulation is a nonlinear degenerate parabolic partial differential problem with non-locally integro-type initial-boundary conditions.

Keywords: Population; Age-structured; Nonlinear; Non-locally; Diffusion; Advection

1. Introduction

As ecological information on life history and habitat characteristics has become more sophisticated, models have become more realistic, and simulation methodology has become more important because analytical analysis of the models has become increasing difficult. This is particularly true for a population in a spatially heterogeneous environment with demographics structured by physiological variables or even variables as rudimentary as age. Understanding the dynamics of natural populations requires knowledge of demographic

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structures and their relationship to the physical environment. This paper presents a theory for a mathematical model proposed in [3] of an age dependent population in a bounded heterogeneous spatial environment $\Omega \subset \mathscr{R}^n$ $(n = 1, 2, 3)$. In the model, it is assumed that individuals do not move in the embryonic stage but do move during the older life stage (after the embryonic stage). The age-space structure of the population is described through the distribution function $u(t, a, x)$ where $t > 0$ is time, $a \in [0, A_m]$ is age (A_m is the maximum possible age) and $x \in \Omega$ is the spatial position. The mathematical model can be written as following degenerate parabolic partial differential problem with non-locally integro-type initial-bounded conditions:

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial a} = -\mu u, \quad t \in (0, T], \quad a \in (0, J], \quad x \in \Omega, \tag{1}$$

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial a} + \nabla \cdot (k \nabla u - \mathbf{q} u) = -\mu u, \quad t \in (0, T], \quad a \in (J, A], \quad x \in \Omega, \tag{2}$$

$$u|_{\partial\Omega} = 0, \tag{3}$$

$$u|_{t=0} = g(a, x), \tag{4}$$

$$u|_{a=0} = \int_0^{A_m} \beta u(t, a, x)\, \mathrm{d}a, \tag{5}$$

where $\partial\Omega$ represents the suitably smooth boundary of Ω and ∂v is its outward normal unit; $J \geqslant 0$ is a constant, $k(t, a, x, P) \geqslant 0$ is the diffusion coefficient, $\mathbf{q}(t, a, x, P)$ is the advection coefficient, $\mu(t, a, x, P) \geqslant 0$ is the mortality rate, $\beta(t, a, x, P) \geqslant 0$ is the reproduction rate, and $P(t, x)$ is the total population number at time t and location $x \in \Omega$, that is,

$$P(t, x) = \int_0^{A_m} u(t, a, x)\, \mathrm{d}a. \tag{6}$$

The integral equation (5) is called the renewal equation (cf. [3,12,13]), and describes the reproduction process for the population. Eq. (1) means that individuals do not move at ages $a \in [0, J]$, the embryonic stage; and (2) means that population have a diffusive and advective movements. Noting that (1) and (2) induce two different age structured population models, respectively, we then call problem (1)–(5) a mixed age structured population problem (cf. [3,4,6]).

If we extend k to $\tilde{k}$ as follows

$$\tilde{k} = \begin{cases} 0 & \text{if } a \in [0, J], \\ k & \text{otherwise,} \end{cases} \qquad \tilde{\mathbf{q}} = \begin{cases} 0 & \text{if } a \in [0, J], \\ \mathbf{q} & \text{otherwise,} \end{cases} \tag{7}$$

then we can combine (1) and (2) as the following unified form.

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial a} - \nabla \cdot (\tilde{k} \nabla u - \tilde{\mathbf{q}} u) = -\mu u, \quad t \in [0, T], \quad a \in [0, A_m], \quad x \in \Omega. \tag{8}$$

We define a variational solution for (1)–(5) as follows. Find $u \in L^2((0, T] \times (0, J], L^2(\Omega)) \cap L^2((0, T] \times (J, A_m], V)$ such that

$$(\partial_\tau u, w) + \alpha(\tilde{k}, \tilde{\mathbf{q}}, u, w) = -(\mu u, w), \quad \forall w \in V, \tag{9}$$