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Existence and uniqueness of pseudo-almost periodic solutions to some classes of semilinear differential equations and applications

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Abstract

This paper is concerned with the existence and uniqueness of pseudo-almost periodic solutions to the class of semilinear differential equations of the form

$$u'(t) + Au(t) = f(t, Bu(t)), \quad (*),$$

where $-A$ is the infinitesimal generator of an analytic semigroup acting on a (complex) Banach space $\mathbb{X}$, $B : D(B) \subset \mathbb{X} \mapsto \mathbb{X}$ is a densely defined closed linear operator, and $f : \mathbb{R} \times \mathbb{X} \mapsto \mathbb{X}$ is a jointly continuous function. Under some suitable assumptions on A, B, and f, the existence and uniqueness of a pseudo-almost periodic solution to $(*)$ is obtained.

MSC: 44A35; 42A85; 42A75

Keywords: Almost periodic function; Fractional powers of operators; Pseudo-almost periodic function; Infinitesimal generator of an analytic semigroup; Existence and uniqueness of pseudo-almost periodic solutions; Second-order partial differential equations

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1. Introduction

Let $(\mathbb{X}, \|\cdot\|)$ be a (complex) Banach space. In [8] it was shown that under some suitable conditions there exists a unique *pseudo-almost periodic* solution to the abstract differential equation

$$u'(t) + Au(t) = g(t, u(t)), \quad t \in \mathbb{R}, \tag{1}$$

where $-A$ is the infinitesimal generator of an analytic semigroup acting on $\mathbb{X}$, and $g : \mathbb{R}\times\mathbb{X} \mapsto \mathbb{X}$ is a jointly continuous function.

In this paper we generalize the previous result [8] by studying the existence and uniqueness of pseudo-almost periodic solutions to some classes of semilinear differential equations of the form

$$u'(t) + Au(t) = f(t, Bu(t)), \quad t \in \mathbb{R}, \tag{2}$$

where $-A$ is the infinitesimal generator of an analytic semigroup acting on $\mathbb{X}$, $B : D(B) \subset \mathbb{X} \mapsto \mathbb{X}$ is a densely defined closed linear operator, and $f : \mathbb{R} \times \mathbb{X} \mapsto \mathbb{X}$ is a jointly continuous function.

Under some additional assumptions on A, B, and f, the existence and uniqueness of a pseudo-almost periodic solution to (2) is obtained by combining both fractional powers of linear operators and the classical Banach's fixed-point principle.

To illustrate our main result (Theorem 3.4) we consider the existence and uniqueness of pseudo-almost periodic solutions to two different types of partial differential equations corresponding to the case where B is a bounded linear operator, and the case where B is seen as an unbounded linear operator, respectively. For that, we suppose that the Banach space $(\mathbb{X}, \|\cdot\|) = (L^2(\mathbb{R}), \|.\|_2)$, and consider the linear operators A, B defined by

$$D(A) = \mathbb{H}^2(\mathbb{R}), \qquad Au(\cdot) = -\Delta u(\cdot) = -u''(\cdot)$$

for each $u(\cdot) \in \mathbb{H}^2(\mathbb{R})$, and

$$D(B) = \{u(\cdot) \in L^2(\mathbb{R}) : q(\cdot)u(\cdot) \in L^2(\mathbb{R})\}, \qquad Bu(\cdot) = q(\cdot).u(\cdot)$$

for each $u(\cdot) \in \mathbb{H}^1(\mathbb{R})$ where $q(\cdot)$ is a (Lebesgue) measurable function.

It is routine to check that the existence and uniqueness of pseudo-almost periodic solutions to (2) is now equivalent to the existence and uniqueness of pseudo-almost periodic solutions to the second-order partial differential equation of the form

$$\frac{\partial u}{\partial t}(t, \xi) = \frac{\partial^2 u}{\partial \xi^2}(t, \xi) + f(t, q(t, \xi)u(t, \xi)), \quad t \in \mathbb{R}, \quad \xi \in \mathbb{R}.$$

Now the unbounded case consists of choosing A as above and let B be the derivative operator defined by

$$D(B) = \mathbb{H}^1(\mathbb{R}), \qquad Bu(\cdot) = u'(\cdot)$$

for each $u(\cdot) \in \mathbb{H}^1(\mathbb{R})$.

In this event the corresponding partial differential equation to (2) is given by

$$\frac{\partial u}{\partial t}(t, \xi) = \frac{\partial^2 u}{\partial \xi^2}(t, \xi) + f\left(t, \frac{\partial u}{\partial x}(t, \xi)\right), \quad t \in \mathbb{R}, \quad \xi \in \mathbb{R}.$$