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Distributed delay feedback control of a new butterfly-shaped chaotic system

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ABSTRACT

Delay feedback control has been proven to be one of the effective methods in the domain of continuous chaos control. In this paper, we apply distributed delay as a self-controlling feedback to implement continuous control of a new butterfly-shaped chaotic system. We investigate the local stability and the existence of Hopf bifurcation of this system. The direction and stability of the bifurcating periodic solutions are obtained by using the normal form theory and center manifold theorem. Stability and Hopf bifurcation properties indicate chaos vanishes as the mean time delay reaches a certain value. Finally, some numerical simulations are carried out to show the effectiveness of the theoretical analysis.

1. Introduction

Chaos has been extensively studied since it was originally found by Lorenz [1]. However, in many practical applications, it is well known that chaos is undesirable and needs to be eliminated. Therefore, controlling chaos is quite necessary. In recent decades, chaos control has received a great deal of attention since the pioneering work of Ott et al. [2]. So far many methods have been explored to implement chaos control, including the OGY method, the PC method, active control method, adaptive control method, backstepping design method, impulsive control method, and so forth. These methods, broadly speaking, can be divided into two categories: the nonfeedback control and feedback control. The latter was firstly proposed by Pyragas [3], and was widely used in time-delayed controlling forces for continuous chaotic systems [4]. Indeed, time-delayed feedback is simple and convenient method of controlling chaos, which serves as a powerful tool to control unstable periodic orbits or control unstable steady states [5,6]. It has many potential applications in various fields of science [7,8]. Based on traditional delay feedback control technique, extended delay feedback method has also been explored, which allows one to stabilize unstable periodic orbits over a large domain of parameters. In [9], Guo et al. studied bifurcation behavior in the control of chaos by extended delay feedback.

Inspired by the above work, in this paper, we intend to introduce distributed delay feedback, a more generalization of time-delayed feedback, into a new butterfly-shaped chaotic system proposed by Kim and Chang [10], which is described by the following three-dimensional autonomous system

$$\begin{cases} \dot{x}(t) = a(y(t) - x(t)), \\ \dot{y}(t) = x(t)z(t) + by(t), \\ \dot{z}(t) = -x^2(t) - cz(t), \end{cases} \tag{1}$$

where $a, b, c \in R$ are parameters. System (1) is chaotic when $a = 30$, $b = 15$, $c = 11$, as illustrated in Fig. 1, which has an upside-down shape of chaotic attractor of the family of the Lorenz-like chaotic systems.

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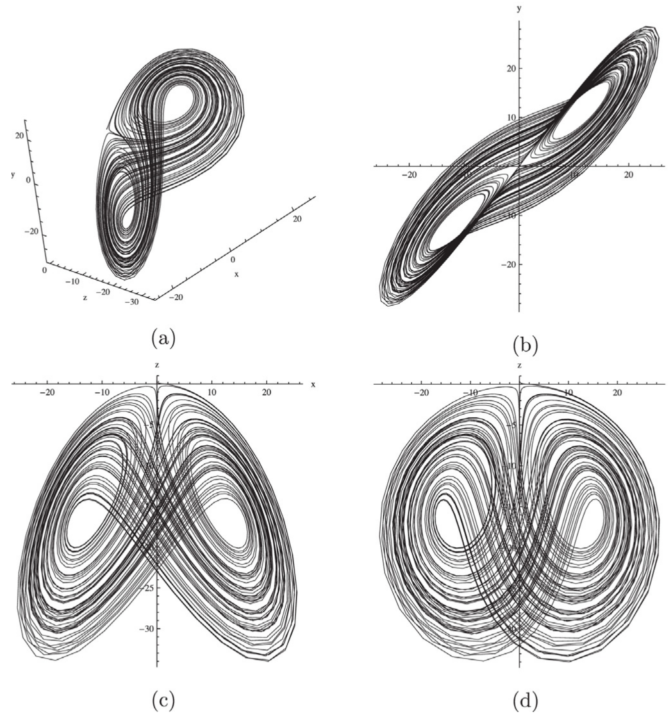

Fig. 1. The attractor of system (1) when $a=30$, $b=15$, $c=11$. (a) Three dimensional view. (b) Projection in the x–y plane. (c) Projection in the x–z plane. (d) Projection in the y–z plane.

In order to apply feedback control, we consider system (1) with continuous distributed delay feedback described by

$$\begin{cases} \dot{x}(t) = a(y(t) - x(t)), \\ \dot{y}(t) = x(t)z(t) + by(t) + M\displaystyle\int_{-\infty}^{0}(y(t) - y(t+s))k(-s)\mathrm{d}s \\ \dot{z}(t) = -x^2(t) - cz(t), \end{cases} \tag{2}$$

where $M \in R$, $k(s)=\gamma e^{-\gamma s}$, $\gamma>0$.

The organization of the rest of this paper is as follows. In the next section, we investigate the local stability and the existence of Hopf bifurcation of the control system (2). In Section 3, the stability of the bifurcating periodic solutions and the direction of the Hopf bifurcation are determined by using the normal form method and the center manifold reduction. In Section 4, some numerical simulations are presented to verify the theoretical analysis. Finally, some concluding remarks are given in Section 5.

2. Local stability and the existence of Hopf bifurcation

In this section, we choose the mean time delay γ as the bifurcation parameter and investigate the local stability of the equilibria and the existence of Hopf bifurcation of system (2), the purpose of which is to explore the effect of distributed delay on the dynamics of system (2).

Clearly, the distributed delay feedback control system (2) and its corresponding system (1) have the same equilibrium points $E_0(0, 0, 0)$, $E_+(\sqrt{bc}, \sqrt{bc}, -b)$, $E_-(-\sqrt{bc}, -\sqrt{bc}, -b)$ if $bc \geq 0$. Without loss of generality, let (x^*, y^*, z^*) be the equilibrium point of system (2), and let $y_1(t)=x(t)-x^*$, $y_2(t)=y(t)-y^*$, $y_3(t)=z(t)-z^*$. Substituting them into system (2) we have

$$\begin{cases} \dot{y}_1(t) = a(y_2(t) - y_1(t)), \\ \dot{y}_2(t) = z*y_1(t) + (b+M)y_2(t) + x*y_3(t) - M\displaystyle\int_{-\infty}^{0} y_2(t+s)k(-s)\mathrm{d}s + y_1(t)y_3(t) \\ \dot{y}_3(t) = -2x*y_1(t) - cy_3(t) - y_1^2(t). \end{cases} \tag{3}$$

Rewrite system (3) as follows:

$$\dot{y}(t) = Ly(t) + \int_{-\infty}^{0} F(s)y(t+s)\mathrm{d}s + H(y), \tag{4}$$

where

$$y(t) = \begin{pmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{pmatrix}, \quad L = \begin{pmatrix} -a & a & 0 \\ z* & b+M & x* \\ -2x* & 0 & -c \end{pmatrix}, \quad F(s) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -Mk(-s) & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad H(y) = \begin{pmatrix} 0 \\ y_1(t)y_3(t) \\ -y_1^2(t) \end{pmatrix}.$$