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A new ranking method based on TOPSIS and possibility theory for multi-attribute decision making problem

Xiaoxia Wang [a,*], Fengbao Yang [a], Hong Wei [b], Lei Zhang [a]

[a] School of Information and Communication Engineering, North University of China, No. 3 Xueyuan Lu, 030051 Taiyuan, Shanxi, China
[b] Computational Vision Group, School of Systems Engineering, University of Reading, Reading RG6 6AY, UK

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ABSTRACT

This paper presents a new ranking method to deal with asymmetric uncertainty information for multi-attribute decision making problem. The uncertainty information of the multi-attribute decision making can be expressed by possibility distribution, and it can be measured to the profit and investment on an organization. In the new ranking method, three coefficients of possibilistic distribution are introduced in order to incorporate a measurement of the uncertainty information firstly. Then the possibilistic mean matrix, possibilistic standard variance matrix and possibilistic skewness matrix are constructed, and the positive ideal solution and the negative ideal solution of decision matrix are determined. Finally, a composite closeness coefficient of each alternative is calculated by separation measure between each alternative in order to rank the preference order of all alternatives and select the most suitable one. Two case studies of tailings dam have been shown to demonstrate the feasibility and efficiency of the presented method.

1. Introduction

Multi-attribute decision making becomes a very important area of operational research and decision science so far [1,2]. It is devoted to solving the most desirable alternative selection problem according to multiple attributes. As one of well-known classical method in the multi-attribute decision making, the technique for order preference by similarity to ideal solution (TOPSIS) has been widely applied in lots of decision areas [3–6], such as system reliability analysis, scheme selection, investment decisions and so on.

Traditionally, researchers dealt with complete certain information and assumed that decision results can be expressed by crisp values, then using TOPSIS method can solving multi-attribute decision making problems under different conditions. However, the monitoring information is often uncertainties and imprecision by nature. Hence, researchers extended the TOPSIS method into uncertainty information. For example, Ye and Li [7] proposed an extended TOPSIS model based on the possibility theory. Shih et al. [8] proposed an extended TOPSIS method for group decision making. Chen and Tsao [9] proposed an interval-valued fuzzy TOPSIS method. Izadikhah [10] used Hamming distance to extend TOPSIS in a fuzzy environment. Vahdani et al. [11] proposed a novel fuzzy modified TOPSIS method which could reflect both subjective judgment and objective information based on the concept of TOPSIS to solve multi-attribute decision making problem with multi-judges and multi-criteria under fuzzy environment. Others extended TOPSIS method from different perspectives [12–14].

In contrast to previous studies on the extensions for TOPSIS method in this paper, we introduce a perfect combination methodology of possibilistic mean, possibilistic variance and possibilistic skewness into the traditional TOPSIS methods by possibility distribution, and present a novel ranking method to deal with the decision problem of asymmetric uncertainty information. As we know, the uncertainty information is always asymmetry, but researchers usually consider it as symmetric information, so as to facilitate decision making and simplifying operation. In addition, if the uncertainty information is asymmetry around the mean, the lower moments and higher moments cannot be consider as equal to quantify the uncertainty on the information. Thus, in the ranking method, the decision maker will select an ideal alternative with possibilistic mean and possibilistic standard variance and possibilistic skewness, in which the possibilistic mean is used as the measurement of information, the possibilistic standard variance is viewed as the measurement of information stability

* Corresponding author. Tel.: +86 13753167491.
E-mail address: wangxiaoxia@nuc.edu.cn (X. Wang).

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and the possibilistic skewnessis considered as the measurement of information symmetry. And in the ranking method the possibilistic mean matrix, possibilistic standard variance matrix and possibilistic skewness matrix are constructed, and the positive ideal solution and the negative ideal solution of decision matrix are determined to compute the composite closeness coefficient of each alternative. The preference order of all alternatives can be ranked according to the composite closeness coefficient of each alternative. Moreover, we also compare the results of our new proposed method and existing methods through an engineering example of tailings dam.

This paper is organized as follows. Section 2 introduces the basic concepts on the possibility theory (such as possibility distribution, possibilistic mean, possibilistic variance and possibilistic skewness) and traditional TOPSIS method. Section 3 presents the new presented ranking method based on TOPSIS and possibilistic theory. Two illustrative examples of tailings dam are considered in Section 4, where the results that are obtained and discussed. Conclusions are given in Section 5.

2. Basic knowledge

2.1. Possibility theory

In practical monitoring of attribute, the monitored data are with uncertainties and imprecision due to limitations of monitoring systems, strong interference of the natural environment and cognitive ability of observer, which is just can be described with possibility theory. So some basic concepts and definitions about possibility theory are introduced at first.

Let us consider an attribute set X', and X'_i is ith attribute for $i = 1, 2, \ldots, k$. To keep things simple, we use $L-R$ adaptive triangular possibility distribution [15] with reference distributions of the each monitoring attribute. The definition of $L-R$ possibility distribution is given as follows:

Definition ($L-R$ adaptive triangular possibility distribution): A possibility distribution π is said to be a $L-R$ possibility distribution, if its distribution function has the following form:

$$\pi(s) = \begin{cases} L\left(\dfrac{a-s}{\alpha}\right)^m & s \in [a-\alpha, a] \\ 1 & s = a \\ R\left(\dfrac{s-\beta}{\beta}\right)^n & s = [a, a+\beta] \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where L and R are the reference functions that define the left and right shapes, respectively. s is the possible value. m and n are the power family parameters of the reference functions, just $m > 0$ and $n > 0$.

In order to maintenance of existing inverse function of $L\left(a-s/\alpha\right)^m$ an $R\left(s-\beta/\beta\right)^n$, we assume $L\left(a-s/\alpha\right)^m = 1-\left(a-s/\alpha\right)^m$ and $R\left(s-\beta/\beta\right)^n = 1-R\left(s-\beta/\beta\right)^n$, so it is easy to see that $L^{-1}(\gamma)^m = a-\alpha(1-\gamma)^{1/m}$ and $R^{-1}(\gamma)^m = a+\beta(1-\gamma)^{1/n}$. When $m = n = 1$, $\pi(s)$ is a triangular possibility distribution; when $m = n \neq 1$, $\pi(s)$ is an adaptive triangular fuzzy number. Let $\pi(s)$ be expressed with the form of interval distribution, namely $\pi(s) = \left[a-\alpha, a, a+\beta\right]_{m,n}$.

Using Definition 1, a γ-level set of possibility distribution π can easily be computed as

$$\forall\gamma \in [0, 1], \quad [\pi]^\gamma = \left[L^{-1}(\gamma), R^{-1}(\gamma)\right] = \left[a-\alpha(1-\gamma)^{1/m}, a+\beta(1-\gamma)^{1/n}\right] \tag{2}$$

According to formula (1) and (2), the possibilistic mean value of possibility distribution π is described as

$$\begin{aligned} M &= \int_0^1 \left(L^{-1}(\gamma)+R^{-1}(\gamma)\right)\gamma \mathrm{d}\gamma = \int_0^1 \left[a-\alpha(1-\gamma)^{1/m}+a+\beta(1-\gamma)^{1/n}\right]\gamma \mathrm{d}\gamma \\ &= a-\left(\frac{m}{2m+1}-\frac{m}{m+1}\right)\cdot\alpha+\left(\frac{n}{2n+1}-\frac{n}{n+1}\right)\cdot\beta \end{aligned} \tag{3}$$

And the possibilistic mean value M will be used as the crisp expected value.

The possibilistic variance of possibility distribution π is described as

$$\begin{aligned} \sigma^2 &= \frac{1}{2}\int_0^1 \left[L^{-1}(\gamma)-R^{-1}(\gamma)\right]^2 \gamma d\gamma = \frac{1}{2}\int_0^1 \left[a-\alpha(1-\gamma)^{1/m}-a-\beta(1-\gamma)^{1/n}\right]^2 \gamma d\gamma \\ &= \frac{\beta^2}{2}\left(\frac{n}{2+n}-\frac{n}{2n+2}\right)+\frac{\alpha^2}{2}\left(\frac{m}{2+m}-\frac{m}{2m+2}\right)+\alpha\beta\left(\frac{mn}{m+n+mn}-\frac{mn}{m+n+2mn}\right) \end{aligned} \tag{4}$$