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Dynamic analysis of a chaotic system

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ABSTRACT

A three-dimensional (3D) autonomous chaotic system is reported in this paper. Some basic properties are analyzed carefully by means of phase portraits, bifurcation diagrams, Lyapunov exponent spectrums and equilibrium points. The remarkable particularity of the system is that it possesses the dynamical properties as constant Lyapunov exponent spectrum and adjustable signal amplitude versus parameters *b*, *d*, and *f*. Further, based on the topological horseshoe theory, the horseshoe chaos in this system is investigated.

1. Introduction

Lorenz system was recognized as the paradigm for chaos investigation since it was found in 1963 when studying the atmospheric convection [1]. With the progress of the chaotic system investigation in diverse fields as fluid mixing, nonlinear circuits, chaotic encryption, chaotic radar, secure communications and power systems protection, scientists have laid themselves out to construct new chaotic systems and analyze their dynamical properties and dynamical behaviors. Many chaotic systems, such as Chen system [2], Liu system [3], Lü system [4], Qi system [5] and so on, have been introduced by developing Lorenz system. However, for these existed chaotic systems, the Lyapunov exponent spectra vary gradually and rang from stable equilibrium points, periodic orbits to chaotic oscillations when changing system parameters continuously.

Recently, a class of chaotic system with constant Lyapunov spectrum based on Colpitts system and Sprott system was proposed [6–11]. For these chaotic systems with constant Lyapunov spectrum, the chaotic behavior is achieved by the piece-wise absolute term, and there exist special amplitude modulation parameters. Amplitude modulation parameters can adjust the signal amplitude of partial [6,11] or whole [7–10] system variables linearly [6–11] or nonlinearly [10,11]. However, the Lyapunov exponent spectrums remain constant. Subsequently, another class of chaotic system with constant Lyapunov spectrum was introduced, whose nonlinear terms are smooth [12–14]. For these chaotic systems [12–14], the amplitude modulation parameters are the coefficient of cross-product term or square term, which can adjust the amplitude of partial [12,14] or whole [13] system variables nonlinearly [12–14]. Because the feature of adjustable amplitude in this kind of system [6–14] can decrease the hardware spending and prevent from increasing the probability of failure in circuit operation, so it can also avoid the influence of the band-limit filter in signal amplification circuit, thus it is a promising type system for application in chaotic radar and chaotic communication.

The topological horseshoe, basic but rigorous chaos theory with symbolic dynamics, is a powerful tool for analyzing chaotic dynamics. One of the pioneering contribution of this theory is the chaos lemma proposed by Kennedy [15,16], which established a topological horseshoe theory for continuous map. Yang obtained another noteworthy result for finding the topological horseshoe in noncontinuous map [17,18]. Latterly, by using several simple results on topological horseshoes, Li presented a new method for seeking horseshoes in dynamical systems [19]. This method has been applied to some practical systems to verify the existence of chaos [12,20,21].

In this paper, we report a 3D autonomous chaotic system. Some basic properties of the presented dynamical system, such as phase portraits, bifurcation diagrams, Lyapunov exponent spectrums and equilibrium points are analyzed deeply. The remarkable particularity of the system is that it possesses the dynamical properties as constant Lyapunov exponent spectrum and adjustable signal amplitude versus parameters *b*, *d*, and *f*. In contrast with the existed results for smooth chaotic system with constant Lyapunov spectrum [12–14], the two amplitude modulation parameters *d* and *f* are the coefficient of cross-product term and square term respectively, but the third amplitude modulation parameter *b* is the coefficient of

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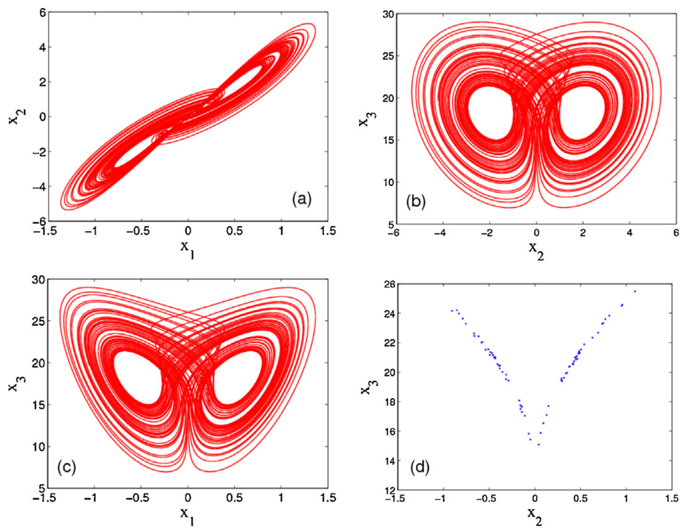

Fig. 1. (a) x_1–x_2 phase portrait; (b) x_2–x_3 phase portrait; (c) x_1–x_3 phase portrait; (d) Poincare mapping.

linear term x_2. Specifically speaking, with the increasing of parameter b, the amplitude of the signals x_1, x_2, x_3 vary by the power function with the index 1/2, −1/2, −1, respectively; with the increasing of parameter d, the amplitude of the signals x_1, x_2, x_3 change by the power function with the index −1/2, −1/2, −1; while the parameter f increases, the amplitude of the signals x_1, x_2 change by the power function with a minus half index, but the third one keeps its amplitude in the same range. Further, by selecting an appropriate cross-section in the chaotic attractor carefully, we find a topological horseshoe of the corresponding Poincare map, thus giving a rigorous verification of the existence of chaos in this system.

2. Chaotic system with constant Lyapunov exponent spectrum

The introduced chaotic system is described by the following ordinary differential equations:

$$\begin{cases} \dot{x}_1 = -ax_1 + bx_2 \\ \dot{x}_2 = cx_2 - dx_1x_3 \\ \dot{x}_3 = -ex_3 + f{x_2}^2 \end{cases} \tag{1}$$

here x_1, x_2, x_3 are the state variables, and a, b, c, d, e, f are the constant parameters with positive values. System (1) reveals chaotic behavior with $a = 18, b = 5, c = 10, d = 2, e = 1, f = 5$. The corresponding chaotic phase diagrams and Poincare mapping on plane $x_1 = 0$ are depicted in Fig. 1. It appears from Fig. 1 that the reported system displays abundantly complicated behaviors of chaotic dynamics.

3. Complex dynamics analysis of 3D system by varying different parameters

Analyses show that this system exhibits complex dynamical behaviors by varying each parameter in a wide range, including constant Lyapunov exponent spectrum, amplitude adjuster and period doubling cascade, which is expatiated below.

3.1. Constant Lyapunov exponent spectrums by varying parameters b, d, f

Let parameters b, d, f vary in some region respectively, but the values of other parameters are chosen as in Section 2. The corresponding Lyapunov exponent spectrums for system (1) are displayed in Fig. 2. It's shown from Fig. 2(a)–(c), the Lyapunov exponent spectrums keep invariable when parameters b, d, f vary in the range [0,6] respectively, and the evolutions of the system flow are always chaotic.

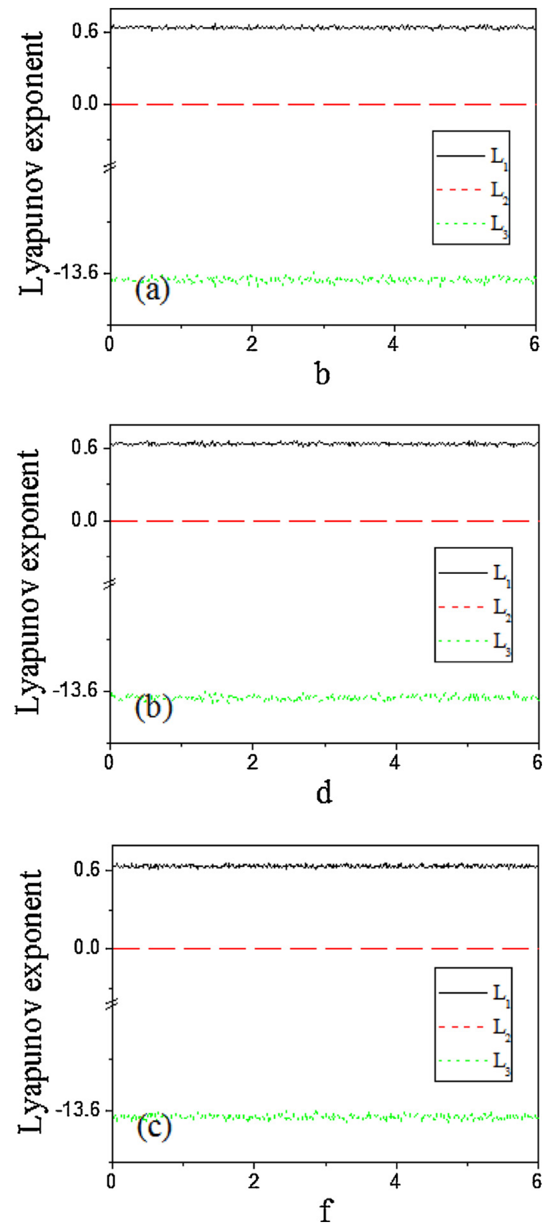

Fig. 2. Lyapunov exponent spectrums of system (1) versus parameters b, d, f.

One can easily get three equilibrium points of system (1), as

$$P_0(0,0,0), \quad P_+\left(\sqrt{\frac{bce}{adf}}, \sqrt{\frac{ace}{bdf}}, \frac{ac}{bd}\right),$$
$$P_-\left(-\sqrt{\frac{bce}{adf}}, -\sqrt{\frac{ace}{bdf}}, \frac{ac}{bd}\right) \tag{2}$$

The corresponding Jacobian matrix of system (1) evaluated at equilibrium point (x_{10}, x_{20}, x_{30}) is depicted by

$$J = \begin{bmatrix} -a & b & 0 \\ -dx_{30} & c & -dx_{10} \\ 0 & 2fx_{20} & -e \end{bmatrix} \tag{3}$$