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Propagation of coupled super-Gaussian beam pairs in strong nonlocal media

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ABSTRACT

The propagation of two mutually incoherent coupled super-Gaussian (SG) beam pairs in strong nonlocal media was studied by variational approach and numerical simulation. For forming a SG vector solitary wave, the total initial power must be equal to the critical power and the ratio of the two beam widths should be equal to a certain value. The numerical results show that the normalized critical power is a monotonically increase function of the order of SG solitary wave. Therefore, the phase shift, which is effectively determined by the critical power, will increase quickly with the order of SG solitary wave increasing. Since the phase shift is large for the low-order SG solitary wave and SG beam has some characteristics different with Gaussian beam, this theoretical result maybe has some potential applications value.

1. Introduction

Vector soliton, which includes temporal and spatial vector soliton, has been investigated widely and made a series of achievements. For instance, Rand experimentally observe the propagation and collision of temporal vector soliton in a linearly birefringent optical fiber [1], Zhang studied dissipative vector solitons in a dispersion managed cavity fiber laser with net positive cavity dispersion [2], Wang reported the observation of multi-component spatial vector solitons of four-wave mixing [3]. In recent years, the study on the propagation of spatial vector soliton in nonlocal media has greatly grown. Such as, the formation of dark-bright vector soliton pairs in nonlocal Kerr-type nonlinear medium was analyzed by Lin [4], the existence and stability of two-component vector solitons in nematic liquid crystals was investigated by Xu [5], the propagation of the Polarized vector dark solitons in nonlocal Kree-type self-defocusing media was discussed by Chen [6]. Shen has studied the incoherently coupled two-color Manakov vector solitons which consist of two Gaussian-shaped beams [7] and two hyperbolic secant shaped beams [8]. Kartashov discussed multipole vector soliton in nonlocal media [9]. Two-dimensional nonlocal vector dipole solitons also have been found [10]. However, the (1 + 2)-dimensional strong nonlocal SG vector soliton has not been studied. To the best of our knowledge, only Mishra has discussed the propagation of single SG beams in (1 + 1)-dimensional strong nonlocal media [11].

Furthermore, SG beam has some different characteristics. For instance, it easy to be identified by detector owe to its flat top and also easy to be accommodated more numbers of pulses in less space because of its narrow tail [11]. In recent years, the application of SG beam has been found in several fields, such as McLaren shows that trapping strength can be tuned continuously by adjusting the order of a SG beam without the need for power adjustment of the laser. Therefore, the study on the propagation of two mutually incoherent SG beams in strong nonlocal media maybe has actual significance.

It is worth noting that the variational approach in nonlinear is uncontrollable (i.e. the accuracy is not predictable) and only provides an approximate solution. However, it can be viewed as a qualitative argument for understanding the possible numerical finding. In this paper, we obtain the critical power and large phase shift of SG solitary wave by variational approach and find it according with the numerical results well.

2. Theoretical model

The propagation of two mutually incoherent coupled beam pairs in (1 + 1)-dimensional nonlocal media is satisfied the following coupled equations [8–12].

$$i\frac{\partial\psi_1}{\partial z}+\mu\frac{\partial^2\psi_1}{\partial x^2}+\rho\psi_1\int_{-\infty}^{\infty}R(x-x')[|\psi_1(x',z)|^2+|\psi_2(x',z)|^2]dx'=0, \tag{1a}$$

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$$i\frac{\partial\psi_2}{\partial z}+\mu\frac{\partial^2\psi_2}{\partial x^2}+\rho\psi_2\int_{-\infty}^{\infty}R(x-x')[|\psi_2(x',z)|^2+|\psi_1(x',z)|^2]dx'=0, \tag{1b}$$

where ψ_i $(i=1,2)$ are the optical beams, $\mu=1/2k$, $\rho=k\eta$, k is the wave number in the media without nonlinearity, η is a material constant and R is a symmetrical real nonlinear spatial response function of the media.

The Lagrange density equation, which corresponding to Eqs. (1a) and (1b), is given as follow

$$\begin{aligned}L=&\frac{i}{2}\left(\psi_1^*\frac{\partial\psi_1}{\partial z}-\psi_1\frac{\partial\psi_1^*}{\partial z}\right)-\mu\left|\frac{\partial\psi_1}{\partial x}\right|^2\\&+\frac{1}{2}\rho|\psi_1|^2\int_{-\infty}^{+\infty}R(x-x')[|\psi_1(x',z)|^2+|\psi_2(x',z)|^2]dx'\\&+\frac{i}{2}\left(\psi_2^*\frac{\partial\psi_2}{\partial z}-\psi_2\frac{\partial\psi_2^*}{\partial z}\right)-\mu\left|\frac{\partial\psi_2}{\partial x}\right|^2\\&+\frac{1}{2}\rho|\psi_2|^2\int_{-\infty}^{+\infty}R(x-x')[|\psi_2(x',z)|^2+|\psi_1(x',z)|^2]dx',\end{aligned}\tag{2}$$

Here we look for solutions to Eq. (2) in SG-shaped

$$\psi_1(x,z)=A_1(z)\exp\left[ic_1(z)x^2+i\theta_1(z)-\frac{1}{2}\left[\frac{x}{a_1(z)}\right]^{2m}\right],\tag{3a}$$

$$\psi_2(x,z)=A_2(z)\exp\left[ic_2(z)x^2+i\theta_2(z)-\frac{1}{2}\left[\frac{x}{a_2(z)}\right]^{2n}\right],\tag{3b}$$

where $A_i(z)\,(i=1,2)$ are the amplitudes, $\theta_i(z)$ are the phases of complex amplitudes, $c_i(z)$ are the phase-front curvatures of the beams, $a_i(z)$ represent the beam widths and m, n stand for the order of the SG optical beams.

In strong nonlocal media, since the characteristic width of the response function is larger than the initial beam width, the response function can be expanded twice and reduced as follow [13–15]

$$R(x-x')\approx R_0-\frac{1}{2}\gamma(x-x')^2,\tag{4}$$

where $R_0=R(0,0)$, $\gamma=-R^{(2,0)}(0,0)$ [13,14]. By inserting the trial function (3a), (3b) and Eq. (4) into Eq. (2) and integrating over x, we obtain the average Lagrange which only depends on the parameters of beams

$$\begin{aligned}L=&-\frac{2}{3}A_1^2a_1^3\Gamma_{m2}\frac{dc_1}{dz}-2A_1^2a_1\Gamma_{m1}\frac{d\theta_1}{dz}-A_1^2m\Gamma_{m3}\frac{\mu}{a_1}\\&-A_1^2\mu\frac{8}{3}c_1^2a_1^3\Gamma_{m2}+2R_0\rho\Gamma_{m1}^2A_1^4a_1^2-\frac{2}{3}\rho\gamma\Gamma_{m1}\Gamma_{m2}A_1^4a_1^4\\&-\frac{2}{3}A_2^2a_2^3\Gamma_{n2}\frac{dc_2}{dz}-2A_2^2a_2\Gamma_{n1}\frac{d\theta_2}{dz}-A_2^2n\Gamma_{n3}\frac{\mu}{a_2}\\&-A_2^2\mu\frac{8}{3}c_2^2a_2^3\Gamma_{n2}+2R_0\rho\Gamma_{n1}^2A_2^4a_2^2-\frac{2}{3}\rho\gamma\Gamma_{n1}\Gamma_{n2}A_2^4a_2^4\\&+4\rho R_0\Gamma_{m1}\Gamma_{n1}A_1^2a_1A_2^2a_2-\frac{2}{3}\rho\gamma\Gamma_{m1}A_1^2a_1\Gamma_{n2}A_2^2a_2^3\\&-\frac{2}{3}\rho\gamma\Gamma_{n1}A_2^2a_2\Gamma_{m2}A_1^2a_1^3,\end{aligned}\tag{5}$$

where $\Gamma(x)$ is the Gamma function, concretely, $\Gamma_{m1}=\Gamma(1+1/2m)$, $\Gamma_{m2}=\Gamma(1+3/2m)$, $\Gamma_{m3}=\Gamma(2-1/2m)$, $\Gamma_{n1}=\Gamma(1+1/2n)$, $\Gamma_{n2}=\Gamma(1+3/2n)$, $\Gamma_{n3}=\Gamma(2-1/2n)$. The evolution equations for the parameters of the two beams can be obtained based on the variational principle

$$A_1^2a_1=A_{10}^2a_{10}=\frac{P_{10}}{2\Gamma_{m1}},\tag{6a}$$

$$\frac{da_1}{dz}-4\mu_1c_1a_1=0,\tag{6b}$$

$$\frac{dc_1}{dz}=\frac{3m\Gamma_{m3}\mu}{2\Gamma_{m2}a_1^4}-4\mu c_1^2-\rho\gamma A_1^2a_1\Gamma_{m1}-\rho\gamma A_2^2a_2\Gamma_{n1},\tag{6c}$$

$$\begin{aligned}\frac{d\theta_1}{dz}=&-\mu\frac{m}{a_1^2}\frac{\Gamma_{m3}}{\Gamma_{m1}}+2\rho R_0a_1A_1^2\Gamma_{m1}-\frac{1}{3}\rho\gamma A_1^2a_1^3\Gamma_{m2}+2\rho R_0a_2A_2^2\Gamma_{n1}\\&-\frac{1}{3}\rho\gamma A_2^2a_2^3\Gamma_{n2},\end{aligned}\tag{6d}$$

$$A_2^2a_2=A_{20}^2a_{20}=\frac{P_{20}}{2\Gamma_{n1}},\tag{6e}$$

$$\frac{da_2}{dz}-4\mu_2c_2a_2=0,\tag{6f}$$

$$\frac{dc_2}{dz}=\frac{3n\Gamma_{n3}\mu}{2\Gamma_{n2}a_2^4}-4\mu c_2^2-\rho\gamma A_2^2a_2\Gamma_{n1}-\rho\gamma A_1^2a_2\Gamma_{m1},\tag{6g}$$

$$\begin{aligned}\frac{d\theta_2}{dz}=&-\mu\frac{n}{a_2^2}\frac{\Gamma_{n3}}{\Gamma_{n1}}+2\rho R_0a_2A_2^2\Gamma_{n1}-\frac{1}{3}\rho\gamma A_2^2a_2^3\Gamma_{n2}+2\rho R_0a_1A_1^2\Gamma_{m1}\\&-\frac{1}{3}\rho\gamma A_1^2a_1^3\Gamma_{m2},\end{aligned}\tag{6h}$$

where P_{i0} $(i=1,2)$ are the initial powers, A_{i0} and a_{i0} represent the initial amplitudes and beam widths, respectively. The evolution equations of beam widths are obtained by combining Eqs. (6b), (6c) and (6f), (6g)

$$\frac{d^2a_1}{dz^2}=\frac{6m\Gamma_{m3}\mu^2}{\Gamma_{m2}a_1^3}-2\mu\rho\gamma a_1(P_{10}+P_{20}),\tag{7a}$$

$$\frac{d^2a_2}{dz^2}=\frac{6n\Gamma_{n3}\mu^2}{\Gamma_{n2}a_2^3}-2\mu\rho\gamma a_2(P_{10}+P_{20}),\tag{7b}$$

By setting $d^2a_i/dz_i^2|_{z=0}=0\,(i=1,2)$, we obtain the critical power of the vector soliton

$$P_{c1}=\frac{3m\Gamma_{m3}\mu}{\Gamma_{m2}a_{10}^4\rho\gamma},\tag{8a}$$

$$P_{c2}=\frac{3n\Gamma_{n3}\mu}{\Gamma_{n2}a_{20}^4\rho\gamma},\tag{8b}$$

For such a vector soliton to exist, the total initial power should be equal to the critical powers, namely $P_0=(P_{10}+P_{20})=P_{c1}=P_{c2}$. Therefore we obtain that the ratio of the initial beam widths should be equal to a certain value which depends on the orders of the two SG beams

$$\frac{a_{20}}{a_{10}}=\sqrt[4]{\frac{n\Gamma_{n3}\Gamma_{m2}}{m\Gamma_{n2}\Gamma_{m3}}}.\tag{9}$$

3. The numerical results

Assuming the response function is Gaussian-shaped $R(x,y)=(1/\pi\sigma^2)\exp[-(x^2+y^2)/\sigma^2]$, we can employ the split-step Fourier method to simulate the propagation of two mutually incoherent coupled SG beam pairs in strong nonlocal media.

Where $X=x/a_{10}$, $y_i=a_i/a_{10}$ $(i=1,2)$ are the normalized beam widths, $Z=z/ka_{10}{}^2$ is the normalized propagation distance.

Fig. 1(a) and Fig. 2(a) depict the normalized intensity distribution and widths of the Gaussian beams which propagate in strong