Optik 126 (2015) 5134–5138

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Launching of the new world of geometrical optics

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ARTICLE INFO

Article history:
Received 22 October 2014
Accepted 9 September 2015

Keywords:
Geometrical optics
Reflection
Refraction
Vector algebra
Cross product

ABSTRACT

This paper reports on the discovery of the most unambiguous generalized vectorial laws of reflection and refraction along with the launching of a new world of geometrical optics with a lot of novel interesting physical insights to optical phenomena.

1. Introduction

Geometrical Optics is a branch of Classical Optics in which optical phenomena are studied on the basis of a few well established laws with simultaneous use of geometrical and analytical methods. It has been reported in [16] that the traditional definition of each of the angles of incidence, reflection and refraction [1–9] is ambiguous on account of having no rationality with the fundamental definition of angle in Geometry, there by leading the traditional laws of reflection and refraction [1–9] to be ambiguous as well.

With a view to getting rid of the ambiguity present in the traditional definition of each of the angles of incidence, reflection and refraction, unambiguous definition of each of the aforesaid three angles has been offered in [16]. Making use of the unambiguous definition of each of the angles of incidence, reflection and refraction, the unambiguous generalized vectorial laws of reflection and refraction have also been developed in [16] to address the two typical problems considered in [16]. But it must be noted that, on account of the lack of specification of appropriate lower and upper bounds, the unambiguous definition of each of the aforesaid three angles offered in [16] is incomplete and hence still remains ambiguous. As a result, the generalized vectorial laws of reflection and refraction developed in [16] are also ambiguous. The same type of ambiguity also exists in each of the equivalent vector forms of reflection and refraction laws available in the traditional literature [10–15].

In view of above, there is an urgent need to refine first the definition of each of the unambiguous angles of incidence, reflection and refraction offered in [16] by incorporation of well defined realistic lower and upper bounds of each of the aforesaid three angles and then to give birth to the most unambiguous statement of each of the generalized vectorial laws of reflection and refraction.

The discovery of the most unambiguous generalized vectorial laws of reflection and refraction has been reported in this paper along with the incorporation of the refined unambiguous definition of each of the angles of incidence, reflection and refraction. The incorporation of the refined unambiguous definition of each of the angles of incidence, reflection and refraction along with the discovery of the most unambiguous generalized vectorial laws of reflection and refraction ultimately gives birth to a new world of Geometrical Optics with a lot of novel interesting physical insights of optical phenomena.

2. Definitions

Refined unambiguous angle of incidence (i)*:* The angle of incidence (i) is the smaller of the angles between the vectors $\mathbf{i}$ and $\mathbf{n}$ subject to the condition that $\pi/2 < i \le \pi$, so long as the case considered is a reflection (or a refraction of light as it passes from a rarer to a denser medium). If however it is a case of refraction as light passes from a denser medium to a rarer medium, the angle i must be bounded by the relation $0 \le i < \pi/2$.

Refined unambiguous angle of reflection (r)*:* The angle of reflection (r) is the smaller of the angles between the vectors $\mathbf{r}$ and $\mathbf{n}$ subject to the condition that $0 \le r < \pi/2$.

Refined unambiguous angle of refraction (R)*:* The angle of refraction (R) is the smaller of the angles between the vectors $\mathbf{n}$ and $\mathbf{R}$

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http://dx.doi.org/10.1016/j.ijleo.2015.09.153
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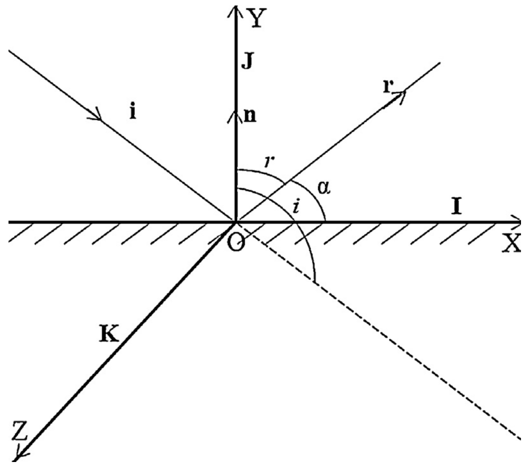

Fig. 1. Diagram showing reflection of light by a plane mirror.

subject to the condition that, $\pi/2 < R \leq \pi$ when the ray of light passes from a rarer medium to a denser medium, or $0 \leq R < \pi/2$ when the ray of light passes from a denser medium to a rarer medium.

3. Ambiguities present in the generalized vectorial laws of reflection and refraction

The generalized vectorial law of reflection reported in [16] or each of its equivalent forms available in traditional literature [10–15] suffers from the lack of specification of well-defined realistic lower and upper bounds of the angles of incidence and reflection, and as a result of which it remains still ambiguous.

In a similar manner, the generalized vectorial law of refraction reported in [16] or each of its equivalent forms available in traditional literature [10–15] suffers from the lack of specification of well-defined realistic lower and upper bounds of the angles of incidence and refraction, and as a result of which it also remains still ambiguous. Furthermore, this law leads to ambiguous result regarding the refractive index of one optical medium with respect to another, computed as the ratio of the sine of the angle of incidence and the sine of the angle of refraction when in particular, both the aforesaid angles correspond to 0°.

In view of above, there is an urgent need of developing the most unambiguous statement of each of the generalized vectorial laws of reflection and refraction and they are being offered next along with theoretical proof of each.

4. The most unambiguous generalized vectorial laws of reflection and refraction

The most unambiguous generalized vectorial law of reflection: If $\mathbf{i}$ and $\mathbf{r}$ represent unit vectors along the directions of incident ray and reflected ray, respectively and if $\mathbf{n}$ represents unit vector along the direction of the positive unit normal to the reflector at the point of incidence then, $\mathbf{n}\times\mathbf{i} = \mathbf{n}\times\mathbf{r}$, where the unambiguous angle of incidence (i) is bounded by the relation, $\pi/2 < i \leq \pi$, and the unambiguous angle of reflection (r) is bounded by the relation $0 \leq r < \pi/2$.

Proof: The proof of the most unambiguous generalized vectorial law of reflection of light can be accomplished on the basis of the principle of conservation of momentum of photon as follows.

Let us consider reflection of a ray of light by a plane mirror as shown in Fig. 1. It clearly follows from Fig. 1 that, in real world, the unambiguous angle of incidence (i) and the unambiguous angle of reflection (r) must be bounded by the relations, $\pi/2 < i \leq \pi$, and $0 \leq r < \pi/2$. Now, assuming the plane of incidence to be the XY plane and considering a right-handed coordinate system, we have from Fig. 1,

$$\mathbf{n}\times\mathbf{i} = (0\mathbf{I}+1\mathbf{J}+0\mathbf{K})\times\left[\left\{\sin(\pi-i)\right\}\mathbf{I}-\left\{\cos(\pi-i)\right\}\mathbf{J}+0\mathbf{K}\right], \quad \text{or,}\quad \mathbf{n}\times\mathbf{i} = (\sin i)(-\mathbf{K}) \tag{1}$$

where $\pi/2 < i \leq \pi$.

Again in this case, we have, $\mathbf{n}\times\mathbf{r} = (0\mathbf{I}+1\mathbf{J}+0\mathbf{K})\times\left\{(\cos\alpha)\mathbf{I}+\left(\cos\beta\right)\mathbf{J}+(\cos\gamma)\mathbf{K}\right\}$, where $\cos\alpha$, $\cos\beta$, and $\cos\gamma$ are the direction cosines of the vector, $\mathbf{r}$ and $0 \leq r < \pi/2$.

or,

$$\mathbf{n}\times\mathbf{r} = (\cos\gamma)\mathbf{I}-(\cos\alpha)\mathbf{K} \tag{2}$$

Now, by applying the principle of conservation of momentum of photon along positive direction of Z-axis, we have,

$$\cos\gamma = 0 \tag{3}$$

Again applying the principle of conservation of momentum of photon along the positive direction of X-axis, we get,

$$\left(\frac{h\nu}{C}\right)\cos(i-90^\circ) = \left(\frac{h\nu}{C}\right)\cos\alpha$$

or,

$$\cos\alpha = \sin i \tag{4}$$

Using relations (3) and (4) we have then from relation (2),

$$\mathbf{n}\times\mathbf{r} = (\sin i)(-\mathbf{K}) \tag{5}$$

where $\pi/2 < i \leq \pi$.

From relations (1) and (5), we have then,

$$\boldsymbol{n}\times\mathbf{i} = \boldsymbol{n}\times\boldsymbol{r},$$

where $\pi/2 < i \leq \pi$, and $0 \leq r < \pi/2$.

Hence proved.

The most unambiguous generalized vectorial law of refraction: If $\mathbf{i}$ and $\mathbf{R}$ represent unit vectors along the directions of the incident ray and refracted ray of a particular colour of light, respectively and if $\mathbf{n}$ represents unit vector along the direction of the positive unit normal to the surface of separation at the point of incidence then, $|(\mathbf{n}\times\mathbf{i})| \sim \mu|(\mathbf{n}\times\mathbf{R})|$, where $(\mathbf{n}\times\mathbf{i})$ and $(\mathbf{n}\times\mathbf{R})$ are like parallel vectors, μ = Refractive index of the second optical medium with respect to the first optical medium for the particular colour of light under consideration, the unambiguous angle of incidence (i) being bounded by the relation, $\pi/2 < i \leq \pi$, when the ray of light passes from a rarer medium to a denser medium, or by the relation, $0 \leq i < \pi/2$, when the ray of light passes from a denser medium to a rarer medium, and the unambiguous angle of refraction (R) being bounded by the relation, $\pi/2 < R \leq \pi$, when the ray of light passes from a rarer medium to a denser medium, or by the relation, $0 \leq R < \pi/2$, when the ray of light passes from a denser medium to a rarer medium.

Proof: Let us first consider the case of refraction of light as it passes from a rarer medium to a denser medium as shown in Fig. 2. It clearly follows from Fig. 2 that, in real world, the unambiguous angle of incidence (i) and the unambiguous angle of refraction (R) must be bounded by the relations, $\pi/2 < i \leq \pi$, and $\pi/2 < R \leq \pi$.

We shall first prove that the vectors, $(\mathbf{n}\times\mathbf{i})$ and $(\mathbf{n}\times\mathbf{R})$ are like parallel vectors. This can be done by applying the principle of conservation of momentum of photon as follows.

Assuming the plane of incidence to be the XY plane and considering a right-handed coordinate system, we have from Fig. 2,

$$\boldsymbol{n}\times\mathbf{i} = (0\mathbf{I}+1\mathbf{J}+0\mathbf{K})\times\left[\left\{\cos(i-90^{\circ})\right\}\mathbf{I}+\left\{-\sin(i-90^{\circ})\right\}\mathbf{J}+0\mathbf{K}\right]$$