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A simple analytical model to study of six wave fiber optical parametric amplifier characteristics

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ABSTRACT

We have made an attempt to present a fairly analytical solution, in order to provide a compact source for such solution for researchers who need to evaluate six wave optical parametric amplifiers (OPA) performance. We have also presented complete derivations, so that the researchers can follow them step by step and thereby gain physical and mathematical insight into the origin of the solutions. We begin by using Maxwell's equations under third nonlinear effects on which fiber OPAs are used to derive complete coupled wave equations. For six waves OPA we also describe some of their fundamental properties resulting from energy conservation, as well as the quantum features of four wave mixing (FWM) interactions. Finally we have presented analytical solutions followed by simulation results for six coupled wave equation for certain conditions.

1. Introduction

In this paper we introduce the six waves fiber optical parametric amplifier (OPA) and a simple analytical model to study their characteristics. We have confined ourselves to situation in which all the waves are launched into the fiber with the same linear state of polarization (SOP) and remain in that state along the entire fiber [1,2]. This allow us to consider a single component of electric field and hence to write scalar equation for it. In Section 2, we first set up the basic OPA equation starting Maxwell's equations for the case of non polarization. In this process we derive an expression for the fiber nonlinearity coefficient γ in terms of waveguide properties and those of the interesting mode. In Section 3, we then proceed with the solution of the six wave OPA equations in a variety of situation for which exact solution are known. The solutions in the absence of loss and pump depletion are relatively simple. They are used extensively in practice as a first approximation to calculating the gain spectra of various fiber OPAs. Solutions in other regimes are more complicated. While it is more difficult to grasp their properties, they can be useful computational tool for obtaining accurate result when the analytical solutions are not applicable.

2. Analytical derivation of the six wave OPA models

In the literature researchers were mostly restricted the attention to the interaction between four waves only, namely, the two pumps, the signal, and the idler. In practice this can be well approximated by placing the pump and the signal approximately with respect to each other and to the zero dispersion wavelengths. Under these circumstances, these four waves carry most of the power over the entire fiber and other waves arising from four-wave mixing are poorly matched and remain at negligible levels [3]. Under certain circumstances, however, some of these other waves may be well phase matched and so may reach level comparable with the signal and the idler. In such situation one must take these waves into account to obtain an adequate description of the situation [4]. An example of such a situation occurs with a two-pump OPA when the signal is close in frequency to one of the pumps as shown in Fig. 1. Then two new waves appear, located at ω_5 and ω_6. The origin can be understood as fallows.

Since the signal is close to pump 1, efficient four-wave mixing (FWM) between these two waves generates the wave at ω_5, which is symmetric to the signal with respect to the pump, i.e. we have $\omega_3+\omega_5=2\omega_1$. Similarly, since idler 1 is close to pump 2, efficient FWM between these two waves generates the wave at ω_6, which is symmetric to idler 1 with respect to pump 2, we have $\omega_4+\omega_6=2\omega_2$. Since $\omega_3+\omega_4=2\omega_c=\omega_1+\omega_2$ we also have $\omega_5+\omega_6=2\omega_c$, this shows that the two new idlers are themselves coupled by the two-pump OPA process. Furthermore, we also have

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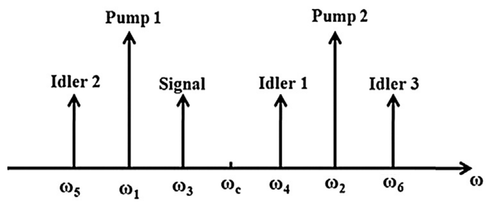

Fig. 1. Frequency assignment for a two-pump OPA, with two sidebands round each pump.

$\omega_4-\omega_5=\omega_6-\omega_3=\omega_2-\omega_1$, which indicates that the signal and the third idler are coupled by a wavelength-exchange type of interaction, as are the first and second idlers [5–7]. Because of all these tight coupling when the signal is close to pump, the signal and the idler can grow together, with gains that are similar. The six wave are coupled by the following equation wave equation [8],

$$\nabla^2 E-\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}=\mu_0\frac{\partial^2 P}{\partial t^2} \tag{1}$$

and we know the relation between E and P as

$$P=\varepsilon_0\left(\chi^{(1)}E+\chi^{(3)}E^3\right) \tag{2}$$

From Eqs. (1) and (2)

$$\nabla^2 E-\frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}=\mu_0\varepsilon_0\chi^{(1)}\frac{\partial^2 E}{\partial t^2}+\mu_0\varepsilon_0\chi^{(3)}\frac{\partial^2 E^3}{\partial t^2} \tag{3}$$

Since $c=\left(1/\sqrt{\mu_0\varepsilon_0}\right)$, $n=\sqrt{1+\chi^{(1)}}$ then Eq. (3) becomes

$$\nabla^2 E=\frac{1+\chi^{(1)}}{c^2}\frac{\partial^2 E}{\partial t^2}+\frac{\chi^{(3)}}{c^2}\frac{\partial^2 E^3}{\partial t^2} \tag{4}$$

$$\nabla^2 E=\frac{n^2}{c^2}\frac{\partial^2 E}{\partial t^2}+\frac{\chi^{(3)}}{c^2}\frac{\partial^2 E^3}{\partial t^2} \tag{5}$$

Computation of last term in above equation is as follow

$$\frac{\partial^2 E^3}{\partial t^2}=\frac{\partial}{\partial t}\left(\frac{\partial E^3}{\partial t}\right)=\frac{\partial}{\partial t}\left(3E^2\frac{\partial E}{\partial t}\right)=3\left[\frac{\partial E}{\partial t}\frac{\partial E^2}{\partial t}+E^2\frac{\partial^2 E}{\partial t^2}\right]$$

Since $E^2=EE^*=\left|E\right|^2$ which is independent of time hence $\partial E^2/\partial t$ term vanishes in above equation than

$$\frac{\partial^2 E^3}{\partial t^2}=3E^2\frac{\partial^2 E}{\partial t^2} \tag{6}$$

We now consider the case where $\chi^{(3)}\neq 0$ and E consist of six frequency components, which satisfy the above conditions. We write the total real electric field as [9–12]

$$E=\frac{1}{2}\sum_{k=1}^{6}\left[B_k(z)\,\psi_k(x,y)\,e^{i\beta_k z-i\omega_k t}+c.c.\right], \tag{7}$$

hence

$$\frac{\partial E}{\partial t}=\frac{1}{2}\sum_{k=1}^{6}\left[-i\omega_k B_k(z)\,\psi_k(x,y)\,e^{i\beta_k z-i\omega_k t}+c.c.\right], \tag{8}$$

$$\frac{\partial^2 E}{\partial t^2}=-\frac{1}{2}\sum_{k=1}^{6}\left[\omega_k^2 B_k(z)\,\psi_k(x,y)\,e^{i\beta_k z-i\omega_k t}+c.c.\right], \tag{9}$$

From Eqs. (5), (6) and (9)

$$\nabla^2 E=\frac{n^2}{c^2}\frac{\partial^2 E}{\partial t^2}+\frac{3E^2\chi^{(3)}}{c^2}\frac{\partial^2 E}{\partial t^2} \tag{10}$$

$$\begin{aligned}\nabla^2 E=&-\frac{n^2}{c^2}\frac{1}{2}\sum_{k=1}^{6}\left[\omega_k^2 B_k(z)\,\psi_k(x,y)\,e^{i\beta_k z-i\omega_k t}+c.c.\right]\\&-\frac{3E^2\chi^{(3)}}{2c^2}\sum_{k=1}^{6}\left[\omega_k^2 B_k(z)\,\psi_k(x,y)\,e^{i\beta_k z-i\omega_k t}+c.c.\right],\end{aligned} \tag{11}$$

Since $K_1=\left(n\omega_1/c\right)$ and $E^2E=EE^*E=[E]^2E^*$ then for given

$$E=\frac{1}{2}\sum_{k=1}^{6}\left[B_k(z)\,\psi_k(x,y)\,e^{i\beta_k z-i\omega_k t}+c.c.\right],$$

Hence

$$\begin{aligned}[E]^2E^*=&\left[\frac{1}{2}\sum_{k=1}^{6}\left[B_k(z)\,\psi_k(x,y)\,e^{i\beta_k z-i\omega_k t}+c.c.\right]\right]^2\\&\times\left[\frac{1}{2}\sum_{k=1}^{6}\left[B_k(z)\,\psi_k(x,y)\,e^{i\beta_k z-i\omega_k t}+c.c.\right]\right]^*\end{aligned} \tag{12}$$

Now by using the following identity

$$\begin{aligned}&(a+b+c+d+e+f)^2\\&=\left[a^2+b^2+c^2+d^2+e^2+f^2+2ab+2ac+2ad+2ae+2af+2bc\right.\\&\quad\left.+2bd+2be+2bf+2cd+2ce+2cf+2de+2df+2ef\right]\end{aligned}$$

It can be shown that

$$\begin{aligned}[E]^2E^*=\ &\frac{1}{8}\left[\left[B_1^2(Z)\,\psi_1^2(x,y)\,e^{2(i\beta_1 z-i\omega_1 t)}\right.\right.\\&+B_2^2(Z)\,\psi_2^2(x,y)\,e^{2(i\beta_2 z-i\omega_2 t)}+B_3^2(Z)\,\psi_3^2(x,y)\,e^{2(i\beta_3 z-i\omega_3 t)}\\&+B_4^2(Z)\,\psi_4^2(x,y)\,e^{2(i\beta_4 z-i\omega 4t)}+B_5^2(Z)\,\psi_1^2(x,y)\,e^{2(i\beta_1 z-i\omega_1 t)}\\&+B_6^2(Z)\,\psi_1^2(x,y)\,e^{2(i\beta_1 z-i\omega_1 t)}\\&+2B_1(Z)\,\psi_1(x,y)\,B_2(Z)\,\psi_2(x,y)\,e^{i(\beta_1+\beta_2)z-i(\omega_1+\omega_2)t}\\&+2B_1(Z)\,\psi_1(x,y)\,B_3(Z)\,\psi_3(x,y)\,e^{i(\beta_1+\beta_3)z-i(\omega_1+\omega_3)t}\\&+2B_1(Z)\,\psi_1(x,y)\,B_4(Z)\,\psi_4(x,y)\,e^{i(\beta_1+\beta_4)z-i(\omega_1+\omega_4)t}\\&+2B_1(Z)\,\psi_1(x,y)\,B_5(Z)\,\psi_5(x,y)\,e^{i(\beta_1+\beta 5)z-i(\omega_1+\omega 5)t}\\&+2B_1(Z)\,\psi_1(x,y)\,B_6(Z)\,\psi_6(x,y)\,e^{i(\beta_1+\beta_6)z-i(\omega_1+\omega_6)t}\\&+2B_2(Z)\,\psi_2(x,y)\,B_3(Z)\,\psi_3(x,y)\,e^{i(\beta_2+\beta_3)z-i(\omega_2+\omega_3)t}\\&+2B_2(Z)\,\psi_2(x,y)\,B_4(Z)\,\psi_4(x,y)\,e^{i(\beta_2+\beta_4)z-i(\omega_2+\omega_4)t}\\&+2B_2(Z)\,\psi_2(x,y)\,B_5(Z)\,\psi_5(x,y)\,e^{i(\beta_2+\beta_5)z-i(\omega_2+\omega_5)t}\\&+2B_2(Z)\,\psi_2(x,y)\,B_6(Z)\,\psi_6(x,y)\,e^{i(\beta_2+\beta_6)z-i(\omega_2+\omega_6)t}\\&+2B_3(Z)\,\psi_3(x,y)\,B_4(Z)\,\psi_4(x,y)\,e^{i(\beta_3+\beta_4)z-i(\omega_3+\omega_4)t}\\&+2B_3(Z)\,\psi_3(x,y)\,B_5(Z)\,\psi_5(x,y)\,e^{i(\beta_3+\beta_5)z-i(\omega_3+\omega_5)t}\\&+2B_4(Z)\,\psi_4(x,y)\,B_5(Z)\,\psi_5(x,y)\,e^{i(\beta_4+\beta_5)z-i(\omega_4+\omega_5)t}\\&+2B_4(Z)\,\psi_4(x,y)\,B_6(Z)\,\psi_6(x,y)\,e^{i(\beta_4+\beta_6)z-i(\omega_4+\omega_6)t}\\&\left.\left.+2B_5(Z)\,\psi_5(x,y)\,B_6(Z)\,\psi_6(x,y)\,e^{i(\beta_5+\beta_6)z-i(\omega_5+\omega_6)t}\right]\right]\\&\times\left[B_1^*(z)\,\psi_1^*(x,y)\,e^{-(i\beta_1 z-i\omega_1 t)}\right.\\&+B_2^*(z)\,\psi_2^*(x,y)\,e^{-(i\beta_2 z-i\omega_2 t)}+B_3^*(z)\,\psi_3^*(x,y)\,e^{-(i\beta_3 z-i\omega_3 t)}\\&+B_4^*(z)\,\psi_4^*(x,y)\,e^{-(i\beta_4 z-i\omega_4 t)}+B_5^*(z)\,\psi_5^*(x,y)\,e^{-(i\beta_5 z-i\omega_5 t)}\\&\left.+B_6^*(z)\,\psi_6^*(x,y)\,e^{-(i\beta_6 z-i\omega_6 t)}\right]\end{aligned}$$