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High fidelity teleportation of coherent superposition states

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ABSTRACT

A high fidelity teleportation scheme for standing wave coherent superposition state is presented. In this scheme, a kind of two-mode coherent entanglement state is taken as quantum entanglement channel, and cavity quantum electrodynamics technique involving the interaction of the atoms with cavity fields is used.

1. Introduction

Recent years quantum teleportation of continuous variable states and atomic states has investigated extensively because it plays very important role in quantum communication [1–11]. A variety of proposals for atomic states teleportation were proposed by using various quantum entanglement channel such as entangled Bell state [2], three-particle entangled state [7], W state [8], three particle GHZ state and four particle GHZ state [10], etc. On the other hand, a variety of teleportation schemes for continuous variable states were also proposed such as the teleportation of coherent states [12–14], squeezed states [15,16], entangled states [17–23], and mesoscopic superposition states [24,25], etc. However, all the teleportation schemes for continuous variable states have one major drawback. This teleportation fidelity is strongly limited. The authors of [26] proposed a teleportation scheme which is based on two-mode single photon entanglement channel. The teleportation fidelity for this scheme is close to 100%, but it still cannot be completely faithful teleportation.

In this paper, we present a new scheme of faithful quantum teleportation for mesoscopic superposition states. In this scheme, the two-mode entangled coherent state is taken as quantum channel, and cavity quantum electrodynamics technique involving the interaction of atoms and cavity fields is used.

2. Theoretical description

Let us consider a three-level atom inside a single-mode light cavity driven by a classical field with frequency ω. The atomic states are denoted by $|g>$, $|e>$, and $|i>$. We assume that the transition frequency between the states $|e>$ and $|i>$ is highly detuned from the cavity frequency. The Hamiltonian of this system is [9] (($\hbar = 1$))

$$H = \frac{\omega_0}{2}\sigma_z + \omega_c a^+ a + g(\sigma_+ a + \sigma_- a^+) + \Omega(\sigma_+ e^{-i\omega t} + \sigma_- e^{i\omega t}) \qquad (1)$$

where $\sigma^+ = |e><g|$, $\sigma^- = |g><e|$, $\sigma^z = |e><e| - |g><g|$, $|e>$, $|g>$ being the excited and ground states of the atom, a^+ and a are the creation and annihilation operators for the cavity mode, g is the atom–cavity coupling strength, and Ω is the Rabi frequency of the classical field. We assume that $\omega_0 = \omega$. Then the interaction Hamiltonian, in the interaction picture, is

$$H_I = e^{-i\delta t} g^* \sigma^+ a + e^{i\delta t} g \sigma^- a^+ + \Omega(\sigma^+ + \sigma^-) \qquad (2)$$

where $\delta = \omega_c - \omega_0$ is the detuning between the atomic transition frequency ($|e> \leftrightarrow |g|>$) and cavity frequency, here we set $\delta = 0$. We make a unitary transformation $H_I' = TH_I T^+ = e^{i\pi\sigma^y/4} H_I e^{-i\pi\sigma^y/4}$; the transformed interaction Hamiltonian is obtained

$$H_I' = \frac{1}{2} g a^+ (\sigma^z - i\sigma^y) + \frac{1}{2} g^* a(\sigma^z + i\sigma^y) + \Omega\sigma^z \qquad (3)$$

Making the rotating wave approximation, which is equivalent to the transformation $e^{-i2\Omega t\sigma_z} H_I' e^{-i2\Omega t\sigma_z}$, the transformed interaction Hamiltonian H_I' becomes

$$H_I' = \frac{1}{2}(g a^+ + g^* a)\sigma^z - \frac{1}{2}(g^* a - g a^+)(\sigma^+ e^{i2\Omega t} - \sigma^- e^{-i2\Omega t}) + \Omega\sigma^z \qquad (4)$$

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Assuming that $2\Omega \gg g$, we can neglect the fast oscillating terms. Then H_I' reduces to

$$H_I' = \frac{1}{2}(ga^+ + g^*a)\sigma^z + \Omega\sigma^z \tag{5}$$

Proceeding to making the anti-transformation, $H_I = T^+ H_I' T$, we can obtain

$$H_I = \frac{1}{2}(ga^+ + g^*a)\sigma^x + \Omega\sigma^x \tag{6}$$

The evolution operator for H_I is represented as

$$U = e^{-itH_I} = e^{-i\Omega t\sigma^x} e^{-i(ga^+ + g^*a)\sigma^x/2} \tag{7}$$

In next section, we will apply Eq. (7) in quantum teleportation of an unknown superposition state.

3. Quantum teleportation for superposition states

We also suppose the teleported state is an unknown superposition state of coherent states

$$|\varphi> = \xi|i\alpha>_1 + \eta|-i\alpha>_1 \tag{8}$$

where a and b are unknown coefficients, $|\xi|^2 + |\eta|^2 + e^{-|\alpha|^2}(\xi\eta^* + \xi^*\eta) = 1$. The two-mode coherent entangled state is taken as the quantum channel

$$|\Pi> = (|i\alpha>_2|i\alpha>_3 + |-i\alpha>_2|-i\alpha>_3)/\sqrt{N} \tag{9}$$

here N is a normalized constant. In our teleportation protocol, in order to obtain high fidelity for the teleported state, we do not use light field joint measurement technique, but use cavity QED technique based on interacting of light field with atoms. We suppose a sender Alice has two atoms 1, 2 with ground states $|g>_1$, $|g>_2$. The initial state of the whole system composed of atoms and cavity fields is given by

$$|\psi>_i = \frac{1}{\sqrt{N}}(\xi|i\alpha>_1 + \eta|-i\alpha>_1) \otimes |g>_1|g>_2 \\ \otimes (|i\alpha>_2|i\alpha>_3 + |-i\alpha>_2|-i\alpha>_3) \tag{10}$$

The whole teleportation protocol is described as follows:

Step 1: Alice sends atoms 1, 2 into the light cavities 1, 2, respectively. The evolution operator U_k of the k-th atoms and the k-th cavity field is described by Eq. (7). We have assumed that coupling parameters Ω, g between atoms and light fields are same. By selecting the interacting time t and the parameters λ, Ω to satisfy the condition $\Omega t = m\pi$, m being a large positive even integer, we can obtain the evolution operator U_k as

$$U_k = \frac{1}{2}\left\{[D_k(i\alpha) + D_k^+(i\alpha)] - [D_k(i\alpha) - D_k^+(i\alpha)]\sigma_k^x\right\}(k = 1, 2) \tag{11}$$

where $D_k(i\alpha) = e^{i\alpha(a_i + a_i^+)}$ is displacement operator of light field in the k-th cavity, $\alpha = gt/2$ (here it is assumed that g is real). According to Eq. (11), we can obtain the state of the whole system as

$$\begin{aligned}
|\psi> = {} & U_1U_2|\psi(0)> + \xi(|0>_1 + |i2\alpha>_1)|g>_1(|0>_2 + |-i2\alpha>_2) \\
& \times |g>_2|-i\alpha>_3 + \eta(|0>_1 + |-i2\alpha>_1) \\
& \times |\,g>_1(|0>_2 + |-i2\alpha>_2)|g>_2|-i\alpha>_3\Big\} \\
& - \Big\{\xi(|0>_1 + |i2\alpha>_1)|g>_1(|i2\alpha>_2 - |0>_2)|e>_2 \\
& \times |i\alpha>_3 + \xi(|0>_1 + |i2\alpha>_1)|g>_1(|0>_2 - |-i2\alpha>_2) \\
& \times |e>_2|-i\alpha>_3 + \eta(|0>_1 + |-i2\alpha>_1) \\
& \times |g>_1(|i2\alpha>_2 - |0>_2)|e>_2|i\alpha>_3 + \eta(|0>_1 + |-i2\alpha>_1) \\
& \times |g>_1(|0>_2 - |-i2\alpha>_2)|e>_2|-i\alpha>_3\Big\} \\
& - \Big\{\xi(|i2\alpha>_1 - |0>_1)|e>_1(|i2\alpha>_2 + |0>_2) \\
& \times |g>_2|i\alpha>_3 + \xi(|i2\alpha>_1 - |0>_1) \\
& \times |e>_1(|0>_2 + |-i2\alpha>_2)|g>_2|-i\alpha>_3 \\
& + \eta(|0>_1 - |-i2\alpha>_1)|e>_1(|i2\alpha>_2 + |0>_2)|g>_2|i\alpha>_3 \\
& + \eta(|0>_1 - |-i2\alpha>_1)|e>_1(|0>_2 + |-i2\alpha>_2) \\
& \times |g>_2|-i\alpha>_3\Big\} + \Big\{\xi(|i2\alpha>_1 - |0>_1) \\
& \times |e>_1(|i2\alpha>_2 - |0>_2)|e>_2\,|i\alpha>_3 \\
& + \xi(|i2\alpha>_1 - |0>_1)|e>_1(|0>_2 - |-i2\alpha>_2)|e>_2| \\
& - i\alpha>_3 + \eta(|0>_1 - |-i2\alpha>_1)| \\
& \times e>_1(|i2\alpha>_2 - |0>_2)|e>_2|i\alpha>_3
\end{aligned} \tag{12}$$

We define the even and odd coherence states for the k-th cavity field and the Bell states for atoms 1 and 2 as

$$|\Phi^+>_k = (|i2\alpha>_k \pm |-i2\alpha>_k)/\sqrt{N_1}(k = 1, 2) \tag{13}$$

$$|\Theta^+>_k = (|gg>_{12} \pm |ee>_{12})/\sqrt{2} \tag{14}$$

$$|\Psi^+>_k = (|ge>_{12} \pm |eg>_{12})/\sqrt{2} \tag{15}$$

where N_1 is normalized constant. By using the even and odd coherence states and the Bell states, Eq. (12) can be represented as

$$\begin{aligned}
|\psi> = {} & |\Theta^+>\Big\{[|0>_1|0>_2 + \frac{N_1}{4}(|\Phi^+>_1 + |\Phi^->_1) \\
& \times (|\Phi^+>_2 + |\Phi^->_2)]\xi|i\alpha>_3 + [|0>_1|0>_2 \\
& + \frac{N_1}{4}(|\Phi^+>_1 - |\Phi^->_1)(|\Phi^+>_2 - |\Phi^->_2)]\eta|-i\alpha>_3 \\
& + \frac{\sqrt{N_1}}{2}[|0>_1(|\Phi^+>_2 - |\Phi^->_2) \\
& + (|\Phi^+>_1 + |\Phi^->_1)|0>_2]\xi|-i\alpha>_3 \\
& + \frac{\sqrt{N_1}}{2}[|0>_1(|\Phi^+>_2 + |\Phi^->_2) \\
& + (|\Phi^+>_1 + |\Phi^->_1)|0>_2]\eta|i\alpha>_3\Big\} + |\Theta^- \\
& > \Big\{[|0>_1|0>_2 + \frac{N_1}{4}(|\Phi^+>_1 + |\Phi^->_1)(|\Phi^+>_2 - |\Phi^->_2)] \\
& \times\xi|-i\alpha>_3[|0>_1|0>_2 + \frac{N_1}{4}(|\Phi^+>_1 - |\Phi^->_1) \\
& \times(|\Phi^+>_2 + |\Phi^->_2)]\eta|i\alpha>_3\frac{\sqrt{N_1}}{2}[|0>_1(|\Phi^+>_2 + |\Phi^->_2) \\
& + (|\Phi^+>_1 + |\Phi^->_1)|0>_2]\xi|i\alpha>_3 \\
& + \frac{\sqrt{N_1}}{2}[|0>_1(|\Phi^+>_2 - |\Phi^->_2) \\
& + (|\Phi^+>_1 - |\Phi^->_1)|0>_2]\eta|-i\alpha>_3\Big\} \\
& + |\Psi^+>\Big\{[|0>_1|0>_2 - \frac{N_1}{4}(|\Phi^+>_1 + |\Phi^->_1) \\
& \times (|\Phi^+>_2 + |\Phi^->_2)]\xi|i\alpha>_3 + [-|0>_1|0>_2 \\
& + \frac{N_1}{4}(|\Phi^+>_1 - |\Phi^->_1)(|\Phi^+>_2 - |\Phi^->_2)]\eta|-i\alpha>_3 \\
& + \frac{\sqrt{N_1}}{2}[|0>_1(|\Phi^+>_2 - |\Phi^->_2) \\
& - (|\Phi^+>_1 + |\Phi^->_1)|0>_2]\xi|-i\alpha>_3 \\
& + \frac{\sqrt{N_1}}{2}[(|\Phi^+>_1 - |\Phi^->_1)|0>_2
\end{aligned}$$