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Annular vortex beams with apertures and their characteristics in the turbulent atmosphere

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ABSTRACT

The analytical formulas for the average intensity distribution of annular vortex beam with aperture in the turbulent atmosphere are derived based on the extended Huygens–Fresnel principle. Under the H-V 5/7 turbulence model, the propagation proprieties of annular vortex beam with aperture are studied, and the influences of the optical topological charge, the propagation distance and the laser wavelength are numerically analyzed.

1. Introduction

Considerable interest has been exhibited in characteristics of vortex beams due to their importance in basic science and some attractive applications, including microlithography, optical trapping, atom detection, optical communication and power coupling [1–7]. Recently, the propagation properties of optical beams carrying phase singularities in the atmospheric turbulence have attracted much attention [8–13]. Gbur and Tyson [14] investigated the propagation of Laguerre–Gaussian vortex beams and suggested that the topological charge could be used as an information carrier in optical communications. Lukin et al. [15] numerically simulated vortex beam propagation through a randomly inhomogeneous medium. However, the aforementioned work was restricted to the aperture unlimited cases and horizontal propagation cases. In actuality, study on annular vortex beam with limited aperture propagation through turbulent atmosphere in a slant path is more important due to the fact that in many applications, dark hollow beam is used as source, aperture of the source is limited by the transmitter and the propagation of laser beam is in a slant path. To the best of our knowledge, study on properties of annular vortex beams with limited aperture propagation through the turbulent atmosphere in a slant path has never been reported.

This paper is organized as follows. First, the general expression of vortex beams with limited aperture is developed in Section 2. Next, the analytical formulas for the average intensity distribution of annular vortex beams with limited aperture propagation through the turbulent atmosphere are derived in Section 3. After that, numerical calculations and results analysis are provided in Section 4. In the end, the conclusions are outlined in Section 5.

2. Annular vortex beam with limited aperture

Optical field of a vortex beam with intensity circular symmetry can be expressed as

$$E(r,\theta)=A(r)e^{il\theta} \tag{1}$$

where (r,θ) denotes the polar coordinates, $A(r)$ denotes the amplitude and l denotes the optical topological charge. By using the Gaussian functions, optical field of a circular dark hollow vortex beam can be expressed as

$$E(r,\theta)=\sum_{n=1}^{N}\frac{(-1)^{n-1}}{N}\binom{N}{n}\left[\exp\left(-\frac{nr^2}{\omega_0^2}\right)-\exp\left(-\frac{nr^2}{\sigma\omega_0^2}\right)\right]\times e^{il\theta} \tag{2}$$

where N is the order of the circular dark hollow function, n represents the offset of the corresponding Gaussian component, $\binom{N}{n}$

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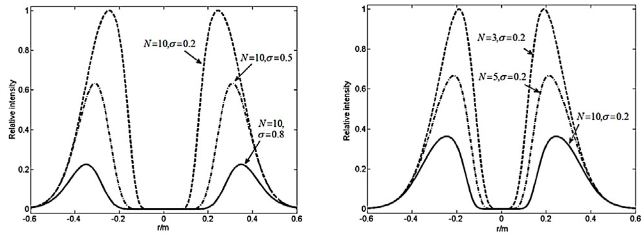

(a) different σ (b) different N

Fig. 1. Beam intensity profiles.

represents the binomial coefficient, σ $(0<\sigma<1)$ is a parameter concerning the circular dark hollow beam, ω_0 is the Gaussian waist width. When $N=1$ and $\sigma\to 0$, Eq. (2) represents a Gaussian vortex beam, and with N increases, Eq. (2) converts to a flattened vortex beam. Relations between beam intensity profile and σ is shown in Fig. 1(a), and relations between beam intensity profile and N is shown in Fig. 1(b), we can get that the area of the dark region across a dark hollow vortex beam increases as σ or N increases.

In optical systems, aperture of the source is commonly limited by the transmitter. The truncation function of the transmitter can be expressed as

$$t(a,b)=\begin{cases}1 & a\le r\le b\\ 0 & \text{else}\end{cases} \tag{3}$$

where a denotes the inner radius and b denotes the outer radius. Generally, the hard-edge aperture function can be expanded as the sum of complex Gaussian functions with finite numbers.

$$t(a,b)=\sum_{w=1}^{M}B_w\times\left[\exp\left(-\frac{C_w}{b^2}r^2\right)-\exp\left(-\frac{C_w}{a^2}r^2\right)\right] \tag{4}$$

where the complex constants B_w and C_w are the expansion coefficients, M is the number of the expansion coefficients. Thus, annular vortex beam with limited aperture can be expressed as

$$\begin{aligned}E_0(r,\theta)=&\sum_{n=1}^{N}\sum_{w=1}^{M}\frac{(-1)^{n-1}B_w}{N}\binom{N}{n}\\&\times\left[\exp\left(-\frac{C_w}{b^2}r^2\right)-\exp\left(-\frac{C_w}{a^2}r^2\right)\right]\\&\times\left[\exp\left(-\frac{nr^2}{\omega_0^2}\right)-\exp\left(-\frac{nr^2}{\sigma\omega_0^2}\right)\right]\times e^{il\theta}\end{aligned} \tag{5}$$

The expression shown in Eq. (5) is a general model, by choosing different ω_0, N, σ and l, we can get vortex beams with different kinds of aperture functions and different optical topological charges.

3. The average intensity distribution in turbulent atmosphere

Schematic diagram of annular vortex beam with limited aperture propagates through the turbulent atmosphere in a slant path is shown in Fig. 2. L denotes the propagation distance, ξ denotes the zenith angle of the propagation path, $E_0(r,\theta)$ denotes the optical field at the source plane $(z=0)$ and $E_1(R,\varphi)$ denotes the optical field at the observation plane $(z=L)$.

By using the extended Huygens–Fresnel principle, the average intensity distribution at the observation plane can be expressed as

$$\begin{aligned}\langle I(\vec{R},L)\rangle=&\frac{k^2}{(2\pi L)^2}\int\int E_0(\vec{r}_1,0)E_0{}^*(\vec{r}_2,0)\\&\times\exp\left\{\frac{ik}{2L}\left[(\vec{R}-\vec{r}_1)^2-(\vec{R}-\vec{r}_2)^2\right]\right\}\\&\times\langle\exp\left[\psi(\vec{R},\vec{r}_1)+\psi^*(\vec{R},\vec{r}_2)\right]\rangle d\vec{r}_1d\vec{r}_2\end{aligned} \tag{6}$$

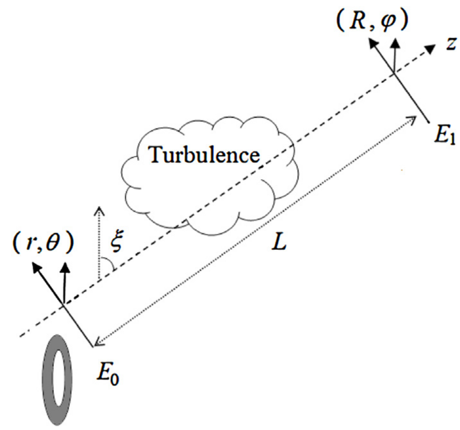

Fig. 2. Schematic diagram of beam propagation through the turbulent atmosphere in a slant path.

where $\vec{R}$ denotes the position vector at the observation plane, $\vec{r}_1$ and $\vec{r}_2$ denote the position vectors at the source plane, k is the wave number, the asterisk denotes the complex conjugation, the $\langle\,\rangle$ indicates the ensemble average over the medium statistics covering the log-amplitude and phase fluctuations due to the turbulent atmosphere, $\psi(\vec{R},\vec{r}_1)$ represents the random part of the complex phase of a spherical wave that propagates from point $(\vec{r}_1,0)$ to point $(\vec{R},L)$. The ensemble average term in Eq. (6) can be expressed as [16]

$$\begin{aligned}\langle\exp[\psi(\vec{R},\vec{r}_1)+\psi^*(\vec{R},\vec{r}_2)]\rangle&=\exp\left[-0.5D_\psi(\vec{r}_1-\vec{r}_2)\right]\\&=\exp\left[-\frac{1}{\rho_0^2}(\vec{r}_1-\vec{r}_2)^2\right]\end{aligned} \tag{7}$$

where D_ψ is the wave structure function, $\rho_0=(0.545\overline{C_n^2}k^2L)^{-3/5}$ is the coherence length of spherical wave propagating in the turbulent medium with $\overline{C_n^2}$ being averaged structure constant of the refractive index along the propagation path.

Inserting Eqs. (5) and (7) into Eq. (6), and perform the related integrals, we can obtain

$$\begin{aligned}&\langle I(R,L)\rangle\\&=\frac{k^2}{(2\pi L)^2}\sum_{n=1}^{N}\sum_{m=1}^{N}\sum_{w=1}^{M}\sum_{v=1}^{M}\frac{(-1)^{n+m}}{N^2}\binom{N}{n}\binom{N}{m}\\&\quad\times B_wB_v^*[(T_1-T_2+T_3-T_4)-(T_1'-T_2'+T_3'-T_4')]\end{aligned} \tag{8}$$

where

$$\begin{aligned}T_1=&\sum_{s=0}^{\infty}\sum_{t=0}^{\infty}\frac{[\Gamma(P_1)]^2\gamma^{2t+s+l}}{(s!)^2t!\,\Gamma(s+t+l)}\left(\frac{k^2R^2}{4L^2}\right)^s\\&\times\left[\exp\left(-\frac{k^2R^2}{4H_{1,1}L^2}\right)F_1\left(P_2;s+1;\frac{k^2R^2}{4H_{1,1}L^2}\right)(H_{1,1})^{-P_1}\right]\\&\times\left[\exp\left(-\frac{k^2R^2}{4H_{1,1}'L^2}\right)F_1\left(P_2;s+1;\frac{k^2R^2}{4H_{1,1}'L^2}\right)(H_{1,1}')^{-P_1}\right.\\&\left.+\exp\left(-\frac{k^2R^2}{4H_{2,2}'L^2}\right)F_1\left(P_2;s+1;\frac{k^2R^2}{4H_{2,2}'L^2}\right)(H_{2,2}')^{-P_1}\right]\end{aligned} \tag{9}$$