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Skeleton extraction based on anisotropic partial differential equation

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ABSTRACT

Skeleton extraction is of great significance for image recognition processes. In this paper, a novel skeleton extraction method is proposed based on the anisotropic diffusion of the gradient vector field. Firstly, the gradient vector is calculated for the original image. Then, the gradient vector field is updated by an anisotropic partial differential equation to strengthen the source point or sink point of the image energy. At last, according to the divergence property of the final gradient vector field, the skeleton of the initial image is obtained. The experimental results demonstrate that the proposed method is robust for noise and can give the exact skeleton for images.

1. Introduction

With the development of the image processing technology, image processing has been widely applied in many fields, such as biomedical [1,2]. As an enabling technology, skeleton extraction of images has been an active topic of research in recent years, both in 2D and 3D [3,4]. Many different methods have been proposed on this topic, such as the topological thinning method [5], the hierarchical approach based on Voronoi diagrams [6,7], the techniques based on the distance maps [8,9] and so on. However, all these methods require pre-processing of the initial images, e.g. enhancement, filtering, segmentation and binarization, and hence the effectiveness of these approaches is significantly impacted by the quality of the initial images and the performance of these pre-processing operations.

Recently, partial differential equation (PDE) has been found as an effective tool for image processing and computer vision [10,11]. The basic idea of PDE-based methods is to set the image to be processed as the initial condition of a PDE with respect to time t, to deform the image by solving the PDE, and to extract the steady state solution of the PDE as the results of the processed image [12]. For the task of skeleton extraction, Yu and Bajaj [13] proposed a method based on anisotropic vector diffusion. In [13] $f(x, y) = \|\nabla G_\sigma(x, y) * I_0(x, y)\|^2$ denotes the edge strength map of the original image $I_0(x, y)$, where $G_\sigma(x, y)$ stands for a Gaussian kernel. With $\nabla f(x, y)$ being the initial conditions of the vector field (u, v), (u, v) is diffused with time by the following:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \mu \cdot div(g(\alpha) \cdot \nabla u) - (u - f_x)\left(f_x^2 + f_y^2\right) \\ \dfrac{\partial v}{\partial t} = \mu \cdot div(g(\alpha) \cdot \nabla v) - (v - f_x)\left(f_x^2 + f_y^2\right) \end{cases} \tag{1}$$

where $\nabla = (\partial/\partial x)\vec{i} + (\partial/\partial y)\vec{j}$, f_x and f_y are the first order spatial derivatives, $g(\cdot)$ is a decreasing function and α is the angle between the central vector and the surrounding vectors. Using non-maximal suppression and double-threshold, the final skeletons are traced from the skeleton strength map [7], which is calculated from the diffused vector field (u, v) and provides a measure on the possibility for each pixel falling on the skeletons.

Direkoglu [14] proposed to diffuse image I in the dominance of direction normal to the feature boundaries while allowing tangential diffusion to contribute slightly, i.e.

$$\frac{\partial I}{\partial t} = cI_{\xi\xi} + I_{\eta\eta} \tag{2}$$

where c is a constant, η denotes the direction normal to the feature boundary (the gradient direction), and ξ is the perpendicular-gradient direction, which is shown in Fig. 1 and given by the following:

$$\xi = \frac{\nabla^{\perp} I}{|\nabla I|} = \frac{(-I_y, I_x)}{\sqrt{I_x^2 + I_y^2}} \quad \eta = \frac{\nabla I}{|\nabla I|} = \frac{(I_x, I_y)}{\sqrt{I_x^2 + I_y^2}} \tag{3}$$

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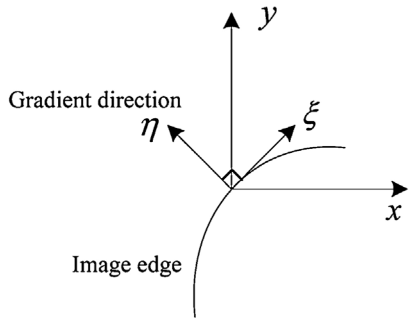

Fig. 1. The inner orthogonal coordinate system.

where $I_{\eta\eta}$ and $I_{\xi\xi}$ are the second order spatial derivatives given as:

$$I_{\xi\xi} = |\nabla I| div\left(\frac{\nabla I}{|\nabla I|}\right) \quad I_{\eta\eta} = \nabla^2 I - |\nabla I| div\left(\frac{\nabla I}{|\nabla I|}\right) \tag{4}$$

Then the skeleton strength map (*SSM*), which provides the likelihood to be a skeleton point, is obtained by computing the mean curvature of level-sets as follows:

$$SSM = \nabla \cdot \left(\frac{\nabla I}{|\nabla I|}\right) \tag{5}$$

Although both methods in [13,14] work directly on the gray-scale image, the quality of the skeleton extracted is unsatisfactory. In [13], Eq. (1) is used to update the two gradient components of the edge strength map of the original image, and the skeleton is extracted based on the updated gradient components. The final result will depend on the initial edge and hence usually exhibits discontinuities. In [14], the initial image is diffused by an anisotropic PDE (i.e., Eq. (2)) first, and noise can be removed by the gray-value diffusion. As the skeleton is obtained based on the smoothed image, it is usually continuous, but many redundant branches can be resulted, especially at the end of the skeletons.

In this paper, a novel method is proposed for the extraction of the skeletons of biomedicine images. The proposed method can be applied directly on to the images, and thus pre-processing is not requires. In contrast to existing works, for example Refs. [13,14], the proposed method is advantageous by skeleton continuity and no branches.

The paper is organized as follows. In Section 2, the novel approach is described in detail. Then, in Section 3, skeleton obtained by the proposed method is presented in comparison with other existing approaches. Finally, conclusion is given in Section 4.

2. Skeleton extraction based on the anisotropic partial differential equation

The main process of the proposed method includes three steps, i.e. initialization of the gradient vector field (GVF) calculation from original image, GVF update by an anisotropic PDE, and skeleton extraction through divergence analysis on the updated GVF.

2.1. The initial gradient vector field

Let $I_0 : R^2 \to R$ represents a gray-level image to be processed, and $I_0(x, y)$ is the gray-level of pixel (x, y). The initial GVF $\vec{F}_0$ can be obtained by

$$\vec{F}_0(x, y) = u_0(x, y)\vec{i} + v_0(x, y)\vec{j} \tag{6}$$

where $u_0(x, y) = \partial I_0(x, y)/\partial x$ and $v_0(x, y) = \partial I_0(x, y)/\partial y$ are the two components of $\vec{F}_0(x, y)$, $\vec{i}$ and $\vec{j}$ are the unit coordinate vectors.

2.2. The diffusion of the gradient vector field

The GVF should be updated with the aim to smooth the noise. This function is similar to various techniques of image smoothing [15]. In order to remove the noise and simultaneously keep the image edges, an anisotropic PDE is introduced to update $u_0(x, y)$ and $v_0(x, y)$ respectively.

Firstly, an inner orthogonal coordinate system is introduced, which is characterized by the local gradient direction (η) and the perpendicular-gradient direction (ξ), respectively (shown in Fig. 1). Then, based on such coordinate system, an anisotropic PDE can be obtained as follows:

$$\frac{\partial I}{\partial t} = I_{\xi\xi} + g(|\nabla G_\sigma * I|)I_{\eta\eta} \quad I(x, y, 0) = I_0(x, y) \tag{7}$$

where $I(x, y)$ is to be updated and I_0 is its initial value. $I_{\xi\xi}$ and $I_{\eta\eta}$ are the second order spatial derivatives along ξ and η respectively and can be expressed as

$$I_{\xi\xi} = \frac{I_{xx}{I_y}^2 - 2I_xI_yI_{xy} + I_{yy}{I_x}^2}{{I_x}^2 + {I_y}^2} \quad I_{\eta\eta} = \frac{I_{xx}{I_x}^2 + 2I_xI_yI_{xy} + I_{yy}{I_y}^2}{{I_x}^2 + {I_y}^2} \tag{8}$$

Also in (7) G_σ is the Gaussian filter, $g(\cdot)$ is a smooth non-increasing function given by $g(s) = k^2/(k^2 + s^2)$ [16] which will influence the diffusion speed, and k is a constant. The diffusion process is carried out based on the following ideas: when (x, y) is located at the edge of the image, $\nabla G_\sigma * I(x, y)$ will exhibit a large value, and $g(|\nabla G_\sigma * I(x, y)|)$ will be a small value, and thus the image edge will be protected; if (x, y) is corrupted by isolated noise, the value of $\nabla G_\sigma * I(x, y)$ will be small, resulting in a large value of $g(|\nabla G_\sigma * I(x, y)|)$, and hence the noise can be smoothed. In other words, fast diffusion will take place around noisy pixels, thus suppressing the noise, and slow diffusion occurs along the local gradient direction, and hence edges can be kept.

Then, according to Eq. (7), u_0 and v_0 can be updated by

$$\begin{cases} \dfrac{\partial u}{\partial t} = u_{\xi\xi} + g(|\nabla G_\sigma * u|)u_{\eta\eta} & u(x, y, 0) = u_0(x, y) \\ \dfrac{\partial v}{\partial t} = v_{\xi\xi} + g(|\nabla G_\sigma * u|)v_{\eta\eta} & v(x, y, 0) = v_0(x, y) \end{cases} \tag{9}$$

and the final GVF is

$$\vec{F}(x, y) = u(x, y)\vec{i} + v(x, y)\vec{j} \tag{10}$$

2.3. Skeleton extraction based on the divergence analysis

Divergence measures the magnitude of the source or sink of a vector field at a given point (x, y) [17]. The divergence of a continuously differentiable vector field $\vec{A}(x, y) = P(x, y)\vec{i} + Q(x, y)\vec{j}$ is:

$$\nabla \cdot \vec{A}(x, y) = \frac{\partial P(x, y)}{\partial x} + \frac{\partial Q(x, y)}{\partial y} \tag{11}$$

When the divergence of a point is greater than zero, i.e. $\nabla \cdot \vec{A}(x, y) > 0$, this point has a positive source which emits flux in the vector field. When the divergence is less than zero, i.e. $\nabla \cdot \vec{A}(x, y) < 0$, this point has a negative source and the flux is sunk in the vector field. When the divergence is equal to zero, i.e. $\nabla \cdot \vec{A}(x, y) = 0$, there is no source at this position.

Let us consider the following gradient vector field:

$$\varphi(x, y) = \nabla \cdot \vec{F}(x, y) = \frac{\partial u(x, y)}{\partial x} + \frac{\partial v(x, y)}{\partial y} \tag{12}$$

The above can be used to determine the source points and sink points, corresponding to the lower-gray region and the higher-gray