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Fresnel diffraction from *N*-apertures: Computer simulation by iterative Fresnel integrals method

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ABSTRACT

The iterative Fresnel integrals method (IFIM) has been applied for the simulation and generation of the complete near-field Fresnel diffraction images created by *N*-apertures for the first time. The simulation can be performed in any PC using a MATLAB program developed by the authors. Necessary formalism was derived for the general *N*-slit problem, and a simulation algorithm was devised for this application. An interesting combination of interference effects with Fresnel diffraction was observed in the simulated images. Transition to the expected Fraunhofer diffraction pattern from Fresnel diffraction for *N*-apertures is also observed in the simulations under the appropriate conditions. Principal maxima of the expected Fraunhofer diffraction were observed at their expected positions, as well as the expected minima and the secondary maxima. The program can serve as a useful tool to study the complex phenomenon of Fresnel diffraction from *N*-apertures, and in addition, to study the near-field Fresnel diffraction from amplitude diffraction gratings.

1. Introduction

In Optics, diffraction is a subject of crucial importance, both from theoretical and experimental points of view [1–3]. Diffraction phenomena can be grossly classified into two types, namely: Fresnel and Fraunhofer. Of these two, the Fresnel or near-field diffraction is relatively more complicated than the Fraunhofer, or far-field diffraction. No exact analytical solution of Fresnel diffraction can be found even in the simplest cases. For example, even for a simple rectangular or circular aperture, no analytical solution can be obtained for Fresnel diffraction. In this case, a solution of the required two-dimensional diffraction integral can be obtained in terms of non-analytical integrals known as the Fresnel cosine and sine integrals [2,3], each involving one-dimensional variables. Even then, visualizing the complicated Fresnel diffraction pattern in terms of the Fresnel integrals can be a difficult task. Usually some visualization tools, such as the Cornu Spiral, are necessary, and even if it used, the complicated diffraction pattern observed is not easy to explain and interpret [2]. It is difficult to predict how the complex and intricate diffraction images will change with a change of experimental conditions, such as aperture–screen distance or aperture dimensions.

We previously introduced [4,5] a new method of calculation of the complete Fresnel diffraction pattern from rectangular-shaped apertures by the iterative Fresnel integrals method (IFIM). The technique has been applied to a single rectangular aperture [5], double apertures and its derivatives [6], triple apertures [7] and the tilted aperture [8]. The usual methods of calculation of Fresnel diffraction from apertures generally employ several types of two-dimensional fast-Fourier transform (FFT)-based algorithms [9–12]. These algorithms are relatively fast and efficient, and can be applied to apertures of any shape. An FFT-based algorithm to calculate Fresnel diffraction by first computing Fraunhofer diffraction image has been proposed recently [13]. Several numerical calculation routines for Fresnel diffraction using FFT have been implemented in the graphics processing unit (GPU) in a computer [14]. While these FFT-based methods are very powerful, using them delegates the entire computation process to the computer, and provides little or almost no insight to the computation process itself [5]. The calculation process is essentially a black box: No existing symmetry properties of the aperture is used to simplify the computation process, nor any attempt is made to a separate the diffraction integral into functions involving one-dimensional variables, as in the case of the Fresnel cosine and sine integrals, as mentioned previously.

In contrast, the IFIM method uses repeated calculation of Fresnel cosine and sine integrals and virtual displacements of the aperture [5]. The method provides a very intuitive method of calculation of the diffraction pattern, and two-dimensional images can be directly generated by the method in any arbitrary experimental

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configurations, such as for any given aperture size, illumination wavelength, and aperture–screen distances. In the IFIM technique, the effect of changing any one parameter (such as aperture–screen distance) on the diffraction image, while all other parameters remain fixed, can be easily observed. The IFIM algorithm was implemented in MATLAB, and the codes can be executed in any PC or laptop computer in a reasonable amount of time, with execution times less than a minute in most cases. Implementation of the codes in other languages, such as Mathematica or MathCad is also possible.

The generalization of the iterative Fresnel integral method to *N*-apertures or slits is not a trivial task. In the calculation of Fresnel integrals for the case of the single, double or triple apertures, the number of Fresnel arrays is fixed. This is not true for the general *N*-slit problem, where the number of Fresnel integrals itself depends on the number of slits *N*, and a simple scaling or expansion of the algorithm to *N* slits in not possible. A different computational approach is needed in this case.

The *N*-slit problem is extremely important in optics, since the *N*-slit can basically serve as an amplitude diffraction grating, which is used in many optical systems, such as spectrographs and spectrometers. Therefore, apart from any theoretical interest, the computation of Fresnel diffraction for the general *N*-slit problem provides a greater insight in the operation of these devices, as diffraction effects can limit the resolution achieved in an optical system.

In this paper, we apply the IFIM technique to a general *N*-slit system. We first derive the basic equation to describe the electric field or intensity due to the *N*-aperture, and then formulate the detailed algorithm for the computation process. The algorithm is then implemented in MATLAB. We present output images from the MATLAB program for typical cases of the *N*-aperture problem. Finally, discussions are made and conclusions are drawn in the final sections.

2. Theory of *N*-slit for the IFIM method

To understand the underlying theory of Fresnel diffraction from an *N*-aperture, let us first consider the simpler case of a single aperture. Light of wavelength λ emitted from a point source *S* is diffracted by a rectangular aperture of dimensions $a \times b$ located at a distance p_0 from it (Fig. 1). The light diffracted from the aperture is observed on the observation plane (or screen) placed a distance q_0 away. The coordinate systems on the aperture and on the image

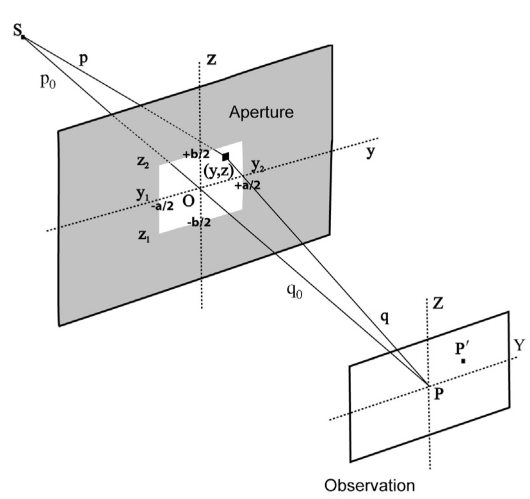

Fig. 1. Basic configuration of Fresnel diffraction from a rectangular aperture of dimension $a \times b$.

planes are chosen, for convenience, to be centered on the optical axis passing through the center of the aperture and orthogonal to it, and are indicated by (*y*,*z*) and (*Y*,*Z*) axes, respectively. The Huygens–Fresnel principle is then used to calculate the total electric field at any given point of the image plane (*Y*,*Z*) by summing up all the contributions (taking into account both amplitude and phase) of all the elementary Huygens wavelets emitted by different area elements inside the rectangular aperture.

By integrating the contributions of the Huygens wavelets over the area d*S* over the area of the whole aperture, it can be shown [2,3] that the total complex electric field at *P* is

$$E_P = \frac{E_u}{2}[C(u)+jS(u)]_{u_1}^{u_2}[C(v)+jS(v)]_{v_1}^{v_2} \tag{1}$$

where E_u is the unobstructed electric field at *P* (i.e. the electric field that would have existed if the aperture were absent), and $C(u)$ and $S(u)$ are the Fresnel cosine and sine integrals, defined by,

$$C(w) = \int_0^w \cos\left(\frac{\pi w'^2}{2}\right) \mathrm{d}w' \quad \text{and} \quad S(w) = \int_0^w \sin\left(\frac{\pi w'^2}{2}\right) \mathrm{d}w'. \tag{2}$$

In the above, *w* represents either of the two dimensionless variables *u* or *v*, being defined as,

$$u = y\sqrt{\frac{2(p_0+q_0)}{\lambda p_0 q_0}}, \qquad v = z\sqrt{\frac{2(p_0+q_0)}{\lambda p_0 q_0}}. \tag{3}$$

The variables *u* and *v* are clearly proportional to the Cartesian coordinates *y* and *z*. Specifically, the limits u_1, u_2, v_1, v_2 correspond to the values of y_1, y_2, z_1, z_2, respectively.

The intensity at *P* is given by the square of E_P, appearing in Eq. (1) i.e., by,

$$I_P = \frac{I_0}{4}\left\{[C(u_2)-C(u_1)]^2+[S(u_2)-S(u_1)]^2\right\}\left\{[C(v_2)-C(v_1)]^2 +[S(v_2)-S(v_1)]^2\right\}. \tag{4}$$

Here I_0 is the unobstructed intensity corresponding to E_u ($I_0 = {E_u}^2$).

Let us now consider the much more complicated case of an *N*-aperture. For simplicity, assume $N = 2n+1$ (odd number of apertures). As before in the single aperture case, we assume that the *N*-aperture system is centered on the *yz* coordinate system, i.e. the origin of the coordinate system *O* is located at the exact center of the *N*-aperture (Fig. 2, shown for $N = 5$). Let *a* be the individual aperture width, *b* be the inter-aperture separation (the center-to-center aperture separation being $(a+b)$), and *c* be the aperture height in the *z*-direction. The two edges of the central aperture of the *N*-aperture system (called aperture **0**) are then located at $y_0 = -a/2$ and $y_0{}' = a/2$, respectively, and the edges of the aperture for the next aperture to the right (called aperture **+1**) are located at $y_1 = a/2 + b$ and $y_1{}' = 3a/2 + \mathrm{b}$, respectively. The next aperture to the right (called aperture **+2**) are located at $y_2 = 3a/2 + 2b$ and $y_2{}' = 5a/2 + 2\mathrm{b}$. Finally the edges of the aperture ***+n*** will be located at $y_n = (2n-1)a/2 + nb$ and $y_n{}' = (2n+1)a/2 + nb$. Similarly, the edges of the first left aperture (aperture −**1**) are located at $y_{-1}{}' = -a/2 - b$ and $y_{-1} = -3a/2 - \mathrm{b}$. Finally, the edges of the aperture ***−n*** will be located at $y_{-n}{}' = -(2n-1)a/2 - nb$ and $y_{-n} = -(2n+1)a/2 - nb$.

Under illumination from the source, the total electric field at the center *P* of the image plane consists of the contributions from all of the $(2n+1)$ apertures. The electric field contribution from the aperture **1** is given, in analogy to Eq. (1), by

$$E_{P_1} = \frac{E_u}{2}[C(u)+jS(u)]_{u_1}^{u_1{}'}[C(v)+jS(v)]_{v}^{v'}. \tag{5}$$