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The threshold gain behavior of random lasing in Kerr-type disordered medium

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ABSTRACT

A model is built to investigate Kerr-type nonlinear effect on the threshold properties of random lasing modes. The numerical simulation shows that compared with the linear case, Kerr-nonlinearity brings about richer modes and higher lasing thresholds.

1. Introduction

The combination of multiple scattering with an optically amplifying media may cause the phenomenon known as random lasing. Unlike a conventional laser, a random laser is an optical device that does not produce a collimated beam of radiation, but instead emits isotropic laser-like light. Such behavior was first theoretically predicted by Letoknov in 1968 [1], who proposed that the incorporation of strong successive scattering events into an optically amplifying media would result in spectrally narrow, spatially diffuse, intense stimulated laser-like light emission. Following this initial work, Lawandy et al. [2] observed laser-like emission in an optically pumped mixture of Rhodamine 640 perchlorate dye in methanol and TiO2 nano-powers. Since that time, random lasers have been reported in diverse systems such as low refractive index polymers [3], rare earth doped dielectric powders [4], high index semiconductor material such as GaAs, GaAsN and ZnO [5–7]. It is noting that Liu et al. [8] have demonstrated both experimentally and numerically that Kerr nonlinearity modifies emission intensity, frequency, pulse duration and size of random lasing modes in disordered medium made of ZnO.

In this work, I devote to investigate the Kerr nonlinear effect on the threshold gain behavior of random lasing in disordered medium. An auxiliary differential equation finite-difference time-domain (ADE-FDTD) method coupled with the rate equation in a four-level energy structure and the equation of motion of polarization was employed. The numerical simulation shows that the central frequencies of the modes for both linear and nonlinear scatterings are only slightly different, whereas threshold of random lasing in nonlinear scattering case is higher than that in the linear case. Further analysis demonstrates that with pumping rates increasing, the frequencies for the linear case remain the same while the excited modes for the nonlinear case shift to shorter wavelength with pumping rates increasing until a minimum value and there exist more excited modes in the whole spectra.

2. Theoretical model

In this section, detailed numerical procedure will be given. For simplicity, the gain and scattering component are separated in our system. The 1D Random medium is a dielectric medium made of two dielectric slabs, as plotted in Fig. 1. The white and black layers describe the gain and scattering layers, respectively. The random thickness and dielectric constant of the gain layers are a_n and $\varepsilon_1 = \varepsilon_0$, respectively. The random thickness is defined as $a_n = a(1 + w\gamma)$, where w is the strength of randomness, $a = 180$ nm and γ is in the range of −0.5 to 0.5. In order to simulate the nonlinear or linear scattering layers, two scattering medium models are selected, respectively. The Kerr nonlinear materials are characterized by the nonlinear polarization while the permittivity of the linear material is chosen as a constant $\varepsilon_2 = 9\varepsilon_0$. The thickness of the scattering layers is a fixed value $b = 300$ nm.

In the Subsection, numerical procedures for the ADE-FDTD analysis of lasing dynamics for gain and scattering medium will be given, respectively.

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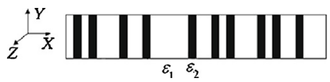

Fig. 1. Schematic illustration of 1D random medium.

2.1. Optical gain material

For optical gain and non-magnetic medium, here we deal with a 1D system, where the time-dependent EM field propagating along the z-axis is simulated using Yee's FDTD algorithm to solve the following Maxwell's equations:

$$\frac{\partial H_y}{\partial x} = \varepsilon_0 \varepsilon_1 \frac{\partial E_z}{\partial t} + \frac{\partial P_{\text{gain}}}{\partial t} \tag{1a}$$

$$\frac{\partial E_z}{\partial x} = \mu_0 \frac{\partial H_y}{\partial t} \tag{1b}$$

where ε_0 and μ_0 are the dielectric permittivity and the magnetic permeability in vacuum, respectively. $\varepsilon_1 = 1$ is the relative electric permittivity of gain medium. P_{gain} is the polarization density, which provides a gain mechanism in the laser system.

For the four-level atomic systems of the gain medium, as shown in Fig. 2, the electron numbers at each energy level, $N_1(z, t)$, $N_2(z, t)$, $N_3(z, t)$ and $N_4(z, t)$ obey the following rate equations.

$$\frac{dN_1}{dt} = \frac{N_2}{\tau_{21}} - W_p N_1 \tag{2a}$$

$$\frac{dN_2}{dt} = \frac{N_3}{\tau_{32}} - \frac{N_2}{\tau_{21}} - \frac{E_z}{\hbar\omega_l} \cdot \frac{dP_{\text{gain}}}{dt} \tag{2b}$$

$$\frac{dN_3}{dt} = \frac{N_4}{\tau_{43}} - \frac{N_3}{\tau_{32}} + \frac{E_z}{\hbar\omega_l} \cdot \frac{dP_{\text{gain}}}{dt} \tag{2c}$$

$$\frac{dN_4}{dt} = -\frac{N_4}{\tau_{43}} + W_p N_1 \tag{2d}$$

where τ_{43} (=1×10^{-13} s), τ_{32} (=1×10^{-10} s), and τ_{21} (=5×10^{-12} s) are the lifetimes at each energy levels, and W_p is the pumping rate of electrons from ground state (level 1) to upper energy level (level 4) and is a controlled variable that should be tuned by the pumping intensity. $\omega_l = (E_3 - E_2)/\hbar = 7.71 \times 10^{14}$ Hz ($\lambda_l = 389$ nm) is the central frequency of emission. The population densities N_i obey the conservation equation $N_T = \sum_{i=1}^{4} N_i = 3.313 \times 10^{24}/\text{m}^3$.

On the basis of the classical electron oscillator (Lorentz) model, one can obtain the following equation of motion of P_{gain} in the presence of an electric field

$$\frac{d^2P(z,t)}{dt^2} + \Delta\omega_l \frac{dP(z,t)}{dt} + \omega_l^2 \frac{dP(z,t)}{dt} = \frac{\gamma_r}{\gamma_c}\frac{e^2}{m}\Delta N(z,t)E(z,t) \tag{3}$$

where $\Delta\omega_l = 1/\tau_{21} + 2/T_2$ is the full width at half-maximum (FWHM) linewidth of the atomic transition, T_2 (=2.14×10^{-14} s) is the mean time between dephasing events, $\Delta N(z,t)$ (=$N_2(z,t) - N_3(z,t)$) is the difference between electron numbers at levels 2 and 3, $\gamma_r = 1/\tau_{32}$, $\gamma_c = (e^2/m)[\omega_l^2/(6\pi\varepsilon_0 c^3)]$ is the classical rate, e is the electron charge, m is the electron mass and c is the speed of light in vacuum.

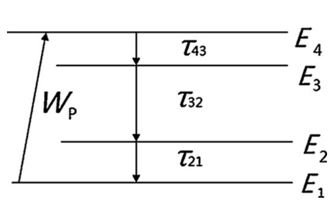

Fig. 2. The scheme of energy levels for gain medium.

2.2. Scattering medium

In the linear and Kerr-nonlinear scattering medium, Maxwell's equations may write

$$\frac{\partial H_y(t,x)}{\partial x} = \frac{\partial D_z(t,x)}{\partial t}, \tag{4a}$$

$$\frac{\partial E_z(t,x)}{\partial x} = \mu_0 \frac{\partial H_y(t,x)}{\partial t}, \tag{4b}$$

$$D_z(t,x) = \varepsilon_\infty \varepsilon_0 E_z(t,x) + P_L(x,t) + P_{NL}(x,t) \tag{4c}$$

where P_L and P_{LN} are the linear and nonlinear polarization density in z direction respectively. Since $P_L = \varepsilon_0 \chi^{(1)} E$, the linear dielectric constant is $\varepsilon_L = \varepsilon_\infty + \chi^{(1)}$. For simplicity, ε_L is selected as a constant $\varepsilon_2 = 9$. Therefore, we may obtain from Eq. (4c):

$$D_z(t,x) = \varepsilon_2 E_z(t,x) + P_{NL}(x,t) \tag{4d}$$

In the scattering layers with Kerr nonlinearity, we introduced the nonlinear polarization density $P_{NL}(x,t) = \varepsilon_0 \chi^{(3)} E(x,t) \int_{-\infty}^{\infty} g(t-\tau) \times |E(x,\tau)|^2 d\tau$, where $\chi^{(3)}$ is the nonlinear coefficient and $g(t-\tau) = (1/\tau_0)\exp(-(t-\tau)/\tau_0)$. τ_0 is the nonlinear response time, is the casual response function. The third-order nonlinear coefficient $\chi^{(3)}$ of ZnO at room temperature range from 10^{-16} to $10^{-14}\,\text{m}^2/\text{V}^2$. In this work, its value is selected as $2 \times 10^{-15}\,\text{m}^2/\text{V}^2$. Note that by setting the values of $\chi^{(3)}$, we may simulate linearity and Kerr nonlinearity in the scattering medium. In order to introduce the nonlinearity into the ADE-FDTD, Ref. [8] gives a new function $G(x,t) = \int_{-\infty}^{\infty} g(t-\tau)|E(x,\tau)|^2 d\tau = (1/\tau_0)\int_0^t e^{-(t-\tau)/\tau_0}|E(x,\tau)|^2 d\tau$. By the use of time differencing for $G(x,t)$, we can obtain

$$\frac{dG(x,t)}{dt} = -\frac{G(x,t)}{\tau_0} + \frac{|E(x,\tau)|^2}{\tau_0} \tag{5}$$

In the above equation, E can be given in the following

$$E(x,t) = [D(x,t) - P_{NL}(x,t)]/\varepsilon_2 \tag{6}$$

where $P_{NL}(x,t) = \chi^{(3)} G|E|^2$. In order to assure numerical stability to the FDTD procedure, the space and time step are $\Delta x = \Delta y = 10$ nm and $\Delta t = \Delta x/(2c) \approx 1.67 \times 10^{-17}$ s, respectively (Fig. 2).

3. Numerical results

For nonlinear scattering medium, we firstly performed the calculation of the spectral intensity under different pump rates, as seen from Fig. 3. Meantime, modes λ_{N0}, λ_{N1}, and λ_{N2} are three long-life lasing modes respectively. Note that each mode is supported by disordered medium. When the pumping rate arrives at low value

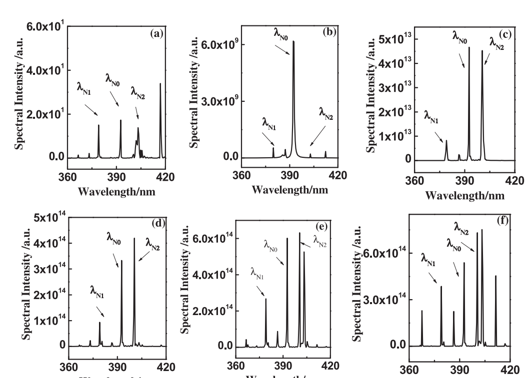

Fig. 3. Output intensity vs. the wavelength for the Kerr-nonlinear scatterings at (a) $W_p = 1 \times 10^9\,\text{s}^{-1}$, (b) $W_p = 1 \times 10^{10}\,\text{s}^{-1}$, (c) $W_p = 1 \times 10^{11}\,\text{s}^{-1}$, (d) $W_p = 1 \times 10^{12}\,\text{s}^{-1}$, (e) $W_p = 1 \times 10^{13}\,\text{s}^{-1}$, and (f) $W_p = 1 \times 10^{14}\,\text{s}^{-1}$.