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Image registration based on Fractional Fourier Transform

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ABSTRACT

Image registration (IR) has been a fundamental task in many image processing applications. Conventional FFT-based methods ignored the spatial information of images, which is also important for IR. So, the Fractional Fourier Transform (FRFT) which contains both spatial and frequency information, has been taken granted in applying to IR scheme. In this paper, a novel method based on the combination of FRFT and conventional phase correlation technique is proposed. Simulation results of the proposed technique prove its better noise immunity than FFT-based method and its superiority than existing methods based on FRFT.

1. Introduction

Image registration (IR) is a fundamental task in image processing to overlay two or more images. The literature [1] roughly divided image registration into two categories. One is based on the spatial domain while the other is based on the transform domain. Methods based on the transform domain have been widely used as its advantages over the traditional spatial registration algorithms. They not only have a low computational complexity and take a fixed period of time in registering any images, but also easy to implement and parallelize.

Some people has described in [2] that, in a shape the dominant information is represented by the variation in its contours and not in its photometric distributions. It is clear to see that spatial information is also important for image registration. However, conventional methods based on Fourier transform ignored the spatial information. As Fractional Fourier Transform (FRFT) [3,4] is a generalization of the conventional Fourier transform and consists of spatial and frequency information, it is natural to extend its use in IR applications. Besides, the use of the FRFT is motivated by the observation in [5] that the optimum FRFT domain for noise elimination may be different from the conventional spatial or frequency domain.

Recently, FRFT has been a preliminary application in image registration [6–8]. Sharma and Joshi carried on an initially research on IR in FRFT domain [6,7]. They regarded IR scheme based on FRFT as extension of time delay estimation. However, data was scarce, so we need to further study this scheme. Then, Zhang succeeded registering medical images in FRFT domain [8]. In Zhang's paper, the registration problem was viewed as a searching of parameters of the transformation model that minimize a cost function in fractional Fourier domain. However, in term of the complexity, Zhang's method was time consuming and the initial value of the parameters had a great influence on the simulation results. These defects limited its application in engineering.

In view of these problems, a novel algorithm, which combines the properties of 2D-FRFT and phase correlation technique [1,3,4] was presented. This method mainly utilizes the translation and rotation properties of FRFT. It not only can get more accurate value of rigid transform parameters than previous FRFT-based methods, but also avoids the iterative operation, which greatly reduces the computation complexity. Simulation results prove the accuracy, noise resistance and simplification of the proposed method.

The rest of the paper is organized as follows. A brief review of the basic theory is given in Section 2. The proposed schemes based on FRFT and complexity analysis are presented in Section 3. Section 4 shows the simulation results. The conclusions are revealed in Section 5.

2. Basic theory

2.1. The Fractional Fourier Transform

Fractional Fourier Transform (FRFT) is a generalization of the conventional Fourier transform (CFT) and has received much attention in recent years [9,10].

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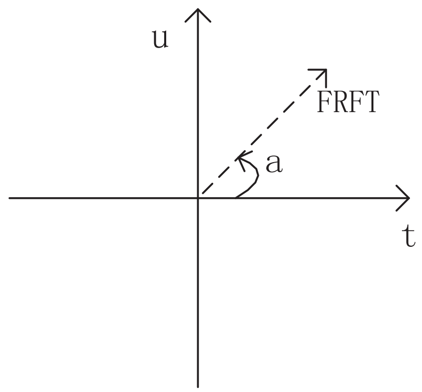

Fig. 1. The Fractional Fourier Transform domain.

$F_\alpha(u)$ defines the FRFT with angle α of a signal $f(t)$,

$$F_\alpha(u) = \int_{-\infty}^{\infty} f(t) K_\alpha(t,u)\, dt, \quad \text{and}$$
$$f(t) = \int_{-\infty}^{\infty} K_\alpha^*(t,u)\, du \tag{1}$$

where the transform kernel $K_\alpha(t,u)$ of the FRFT is given by

$$K_\alpha(t,u) = \begin{cases} \sqrt{\dfrac{1-j\ \cot\alpha}{2\pi}} \exp\{j(t^2+u^2)\cot\ \alpha/2 - jtucsc\alpha\}, & \alpha \neq \pi \\ \delta(t-u), & \alpha = 2n\pi \\ \delta(t+u), & \alpha = (2n+1)\pi \end{cases} \tag{2}$$

Here α indicates the rotation angle of the transformed signal in Wigner domain and * denotes complex conjugation. When $\alpha = \pi/2$, FRFT reduces to Fourier transform, then the u axis is the traditional Fourier axis; when $\alpha = 0$, u axis reduces to the spatial axis. This further confirms that Fractional Fourier Transform contains both spatial and frequency information. The FRFT can be graphically illustrated in Fig. 1.

FRFT has a lot of useful properties including its product and convolution theorems [9,10], which are generalizations of corresponding properties of the DFT. Time translation property of FRFT is like this:

$$f(t-\tau) \leftrightarrow \exp(j\pi\tau^2\ \sin\alpha\cos\alpha)\exp(-j2\pi u\tau\ \sin\alpha)$$
$$F_\alpha(u-\tau\ \cos\alpha) \tag{3}$$

More excellent and comprehensive discussion of FRFT can be found in [5].

2.2. Phase correlation technique

We introduce phase technique here [2]. Let f_1 and f_2 are the two images and $f_2(x,y)$ is translated replica of $f_1(x,y)$

$$f_2(x,y) = f_1(x-x_0,\ y-y_0) \tag{4}$$

Implement Fourier transform on the $f_1(x,y)$ and $f_2(x,y)$, respectively:

$$F_2(u,v) = e^{-j2\pi(ux_0+vy_0)} F_1(u,v)$$

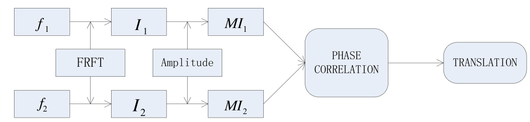

Fig. 2. The block diagram of our proposed methods for translated images.

The cross-power spectrum holds that

$$\frac{F(u,v)G^*(u,v)}{\left|F(u,v)G^*(u,v)\right|} = e^{-j2\pi(ux_0+vy_0)},$$

where G^* is the complex conjugate of G, the shift theorem of Fourier transform guarantees that the phase of the cross-power spectrum is equivalent to the phase difference between images. By taking inverse Fourier transform of the cross-power spectrum, we will get a Dirichlet function, the translation parameter can be simply determined by finding the position of pulse peak.

3. Image registration on FRFT domain

In this section, IR problems for translated and rotated images are introduced respectively. Complexity analysis of the proposed methods will be given at last.

3.1. IR problem for translated images

f_1 and f_2 are the two images and $f_2(x,y)$ is translated replica of $f_1(x,y)$ as shown in (4). The translation property of two dimensions FRFT can be considered an extension of time translation property of one dimension. Then we can get from (3) and (4):

$$I_2(u,v) = I_1(u-x_0\ \cos\alpha, v-y_0\ \cos\beta) e^{j\pi\ \sin\alpha\ \cos\alpha\Delta x^2} e^{j2\pi u\ \sin\beta} e^{j\pi\ \sin\beta\ \cos\beta\Delta y^2} e^{j\pi\ \sin\beta\Delta y}, \tag{5}$$

where $I_1(u,v)$ and $I_2(u,v)$ are the 2D FRFT of image f_1 and f_2, respectively. α and β are the rotation angles of the two dimensions FRFT.

Denote MI_1 and MI_2 are the magnitudes of $I_1(u,v)$ and $I_2(u,v)$. Then, from (5), we have

$$MI_2(u,v) = MI_1(u-x_0\ \cos\alpha, v-y_0\ \cos\beta) = MI_1(u-x_0', v-y_0'), \tag{6}$$

where $x_0' = x_0\ \cos\alpha$, $y_0' = y_0\ \cos\beta$.

From (6) we can get that MI_2 is the translated replica of MI_1, in which the shift parameter becomes (x_0', y_0'). Then the conventional phase correlation technique [2] was used as our similarity measure to estimate (x_0', y_0') in FRFT domain. Finally the translation parameter (x_0, y_0) will be get from formula (6).

The steps to register translated images are shown in Fig. 2.

3.2. IR problem for rotated images

Besides translation, rotation is another kind of rigid transformation. The IR problem for rotated images in FRFT domain is based on the fact that the FRFT of the rotated image by some angle is the rotated version the FRFT of the original image by the same angle [4,11].

$F(u,v)$ is the two dimensional Fractional Fourier Transform of the reference image $f(x,y)$ which could be defined as:

$$F(u,v) = \iint f(x,y) K_{p_1,p_2}(x,y,u,v)\ dx\,dy, \tag{7}$$

where $K_{p_1,p_2}(x,y,u,v) = K_{p_1}(x,u)K_{p_2}(y,v)$.