Optik 126 (2015) 1423–1428

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Performance analysis of multihop communication using generalized gamma fading model

Amandeep Kaur [a,*], Jyoteesh Malhotra [b]

[a] Department of Mathematics, Guru Nanak Dev University College, Jalandhar 144001, Punjab, India
[b] ECE Department, Guru Nanak Dev University Regional Campus, Jalandhar 144001, Punjab, India

ARTICLE INFO

Article history:
Received 27 February 2014
Accepted 9 April 2015

Keywords:
Mellin transformation
Multihop communication
Cascade fading
Generalized gamma
Amount of fading

ABSTRACT

Due to ubiquitous access of personal communication services multiple communication through multihops are needed for exchange of data between source and destination across the wireless network. Recently cascade fading models have gained significance as they can model multihop relay fading channels. Generalized Gamma (GG) model being a versatile, flexible and multiparameter model has been selected here as it can generalizes commonly used fading models such as Weibull, Nakagami-m etc. Exact closed expressions of Probability Density Function (PDF) and Cumulative Distribution Function (CDF) are obtained here for the product of multiple GG random variables (RVs). Based on the derived expressions performance analysis of multihop links has been done here in terms of coefficient of variation (CV), amount of fading (AF) and Spectral efficiency and outage probability. Despite of the exact form, the expressions obtained here are in closed form, tractable and easy to evaluate.

1. Introduction

Wireless is the fastest growing area in the field of communication. Radio signals propagation through wireless channels is a complicated phenomenon characterized by various effects including path loss, multipath fading and shadowing. Except path loss which is distant dependent the other two parameters can be statistically described by various fading models. Depending upon the radio propagation environment and communication scenarios various multipath fading models are available in literature [1]. As the expectations for the performance and reliability of wireless systems have increased, the importance of accurate and simplified channel modeling in system design, evaluation has also increased. Along with the complexity of the channel model comes the complexity of the analytical solution that helps to assess the performance. A versatile wireless channel model which can generate other models for multipath fading and shadowing is three parameters multihop generalized gamma model. Various methods are available in literature for evaluating the PDF and CDF of product of independent random variables. In [2] the expressions for the product of Nakagami-m were obtained in terms of Fox H function. In [3,4] PDF of generalized k channel and Weibull distribution was evaluated using inverse Laplace transforms. In [5] expressions for moment generating function (MGF) of generalized gamma were computed using Fox's H function. In [6] PDF, CDF of generalized gamma are evaluated by applying inverse Laplace transformation to MGF. Hence the previous derivation of expression of PDF involves inverse Laplace transformation of MGF. Also solutions of expressions in terms of Fox's H function are complex as it involves intricate algorithms in evaluation and even it is not guaranteed that it will give stable results. Considering the demerits of the above methods an alternative and efficient method i.e. Mellin Transformation (MT) method has been used to provide simplified and unified performance analysis for generalized gamma channels. The MT method translates exponentials to polynomials so that the product convolution property can be applied for evaluating the expressions for the PDF and CDF of product of independent random variables. The results are obtained in Meijer-G function form as this function is easily available in many scientific software packages such as Maple and Mathematica. This method has also been used in recent past. The expressions for PDF of Gaussian, Rayleigh and other distribution were obtained in [7]. In [8] PDF, MGF and CV of heterogeneous Nakagami–Weibull distribution was derived and in [9] moments and PDF of multiple Rayleigh variates were computed. However an attempt has also been made in [7] to compute the PDF and CDF of generalized gamma but approximate expressions were obtained. Considering the importance of exact form of expressions, Mellin transformation (MT) has been incorporated here for the derivation of closed form expressions of PDF and CDF of product of multiple

* Corresponding author. Tel.: +91 1812465252.
E-mail addresses: hargobind.ji@gmail.com (A. Kaur), jyoteesh@gmail.com (J. Malhotra).

http://dx.doi.org/10.1016/j.ijleo.2015.04.019
0030-4026/

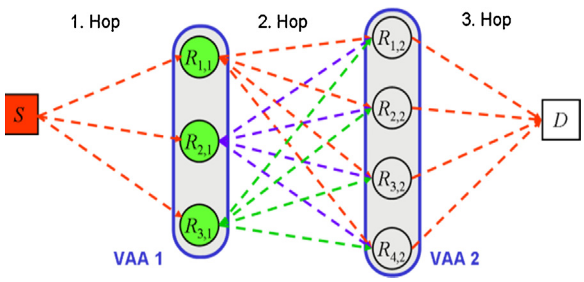

Fig. 1. Propagation through multihop relays.

generalized gamma random variables. The expressions obtained here are in closed form, tractable and easy to evaluate, unlike the earlier intricate expressions obtained using inverse Laplace approach based on MGF.

Further, important performance measures like coefficient of variation (CV), amount of fading (AF), spectral efficiency and outage probability are also derived here. These measures quantify the possible variations in channel gain and SNR from their average values and hence can be used to characterize the receiver performance.

The rest of the paper is organized as follows: Section 2 presents system and channel model, exact closed form expressions of PDF and CDF of product of N generalized gamma model are obtained in Section 3. Performance metrics like CV, AF, spectral efficiency, outage probability are evaluated in Section 4, before the paper is finally concluded in Section 5.

2. System and channel model

The signal leaving the transmitter reaches the receiver after multiple scattering. The most suitable way to model the realistic fading conditions when multiple scattering exists is through cascaded approach. The practical applications of cascade fading models include modeling of wireless signal propagating through multihop relay terminals and keyholes where the communication channel can be described as product of independent random variables. In multihop system several low power relay stations (nodes) are deployed between the source and receiver destination extending the range of wireless network.

Considering multihop wireless transmission (as shown in Fig. 1) which operates over independent but not necessarily identically distributed (n.i.d.) generalized gamma fading channels in the presence of AWGN (Additive White Gaussian Noise). Define the distribution of Y as product of $N \geq 1$ independent but n.i.d. GG random variables X_i, $1 \leq i \leq N$.

$$Y = \prod_{i=1}^{N} X_i$$

where PDF of each RV X_i is given as ([10], Eq. (1))

$$f_{X_i}(x) = \frac{2\upsilon m^m x^{2m\upsilon-1}}{\Gamma(m)\Omega_i^m} \exp\left(-\frac{mx^{2\upsilon}}{\Omega_i}\right) \quad (1)$$

where $\upsilon > 0$ and $m \geq 1/2$ are two shaping parameters (same for all RV's), Ω_i is fading parameter and $\Gamma(.)$ is the gamma function.

The received channel at the destination is given as ([5], Eq. (4))

$$r_d(t) = s(t)\prod\nolimits_{i=1}^{N} X_i A_{i-1} + \sum\nolimits_{i=1}^{N} n_{0,i}(t)\prod\nolimits_{j=i+1}^{N} X_j A_{j-1}$$

with $A_0 = 1$. The instantaneous SNR at the destination terminal can be represented as ([5], Eq. (5)).

$$\gamma_{out} = \frac{\prod_{i=1}^{N} X_i^2 A_{i-1}^2}{\sum_{i=1}^{N} n_{0,i}(t)\prod_{j=i+1}^{N} X_j^2 A_{j-1}^2}$$

where A_i is amplification factor for ith relay and X_i denotes the fading coefficient and $n_{0,i}$ denotes the additive noise at ith relay.

3. Closed form expressions for PDF and CDF for product

In this section PDF and CDF have been derived for the product of N generalized gamma variates using Mellin transform approach. The Mellin transform of PDF $f_X(x)$ is defined as ([11], Eq. (8.2.5))

$$\varphi_X(f_X(x), s) = E(X^{s-1}) = \int_0^{\infty} f(x)x^{s-1}\,dx$$

where $s \in C$ is complex transform variable, $E(.)$ is the expectation operator.

Using (1) the Mellin transform for $f_X(x)$ becomes

$$\varphi_X(s) = \int_0^{\infty} x^{s-1}\frac{2\upsilon m^m x^{2m\upsilon-1}}{\Gamma(m)\Omega_i^m} \exp\left(-\frac{mx^{2\upsilon}}{\Omega_i}\right) dx$$

$$\varphi_X(s) = \frac{\Gamma(m+((s-1)/2\upsilon))}{\Gamma(m)}\left(\frac{\Omega_i}{m}\right)^{(s-1)/2\upsilon}$$

Using product convolution property of Mellin transforms, the Mellin transformation for PDF of product of N generalized gamma variates is:

$$\varphi(f_Y(y), s) = \varphi_Y(s) = \frac{\Gamma^N(m+((s-1)/2\upsilon))}{\Gamma^N(m)}\left(\frac{\prod_{i=1}^{N}\Omega_i}{m^N}\right)^{(s-1)/2\upsilon} \quad (2)$$

The rth order moment of the product is evaluated by replacing $s-1$ by r in (2)

$$E(Y^r)\frac{\Gamma^N(m+(r/2\upsilon))}{\Gamma^N(m)}\left(\frac{\prod_{i=1}^{N}\Omega_i}{m^N}\right)^{r/2\upsilon} \quad (3)$$

The PDF of product is given by inverse Mellin transforms as ([11], Eq (8.2.6))

$$f_Y(y) = \frac{1}{2\pi i}\int_{c-j\infty}^{c+j\infty} y^{-s}\varphi_Y(s)\,ds$$

$$f_Y(y) = \frac{1}{2\pi i}\int_{c-j\infty}^{c+j\infty} y^{-s}\frac{\Gamma^N(m+((s-1)/2\upsilon))}{\Gamma^N(m)}\left(\frac{\prod_{i=1}^{N}\Omega_i}{m^N}\right)^{(s-1)/2\upsilon} ds \quad (4)$$

Substituting $(s-1)/2\upsilon = s'$ the expression (4) reduces to

$$f_Y(y) = \frac{1}{y\Gamma^N(m)2\pi i}\int_{c-j\infty}^{c+j\infty}\left(\frac{m^N}{\prod_{i=1}^{N}\Omega_i}y^{2\upsilon}\right)^{-s'}\Gamma^N(m+s')\,ds' \quad (5)$$

Considering Meijer-G function of the form

$$G_{p,q}^{m,n}\left(y\Big|_{b_1,b_2,\ldots,b_q}^{a_1,a_2,\ldots,a_p}\right) = \frac{1}{2\pi i}\int\frac{\prod_{i=1}^{m}\Gamma(b_i+s)\prod_{i=1}^{n}\Gamma(1-a_i-s)}{\prod_{i=n+1}^{p}\Gamma(a_i+s)\prod_{i=m+1}^{q}\Gamma(1-b_i-s)}y^{-s} \quad (6)$$