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Multichannel sampling theorem for bandpass signals in the linear canonical transform domain

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ABSTRACT

The linear canonical transform (LCT) describes the effect of first-order quadratic phase optical system on a wave field. The classical multichannel sampling theorem for common bandlimited signals has been extended differently to bandlimited signals associated with LCT. However, a practical issue associated with the reconstruction of the original bandpass signal from multichannel samples in LCT domain still remains unresolved. The purpose of this paper is to introduce a practical multichannel sampling theorem for bandpass signals in LCT domain. The sampling expansion which is constructed by the ordinary convolution in the time domain can reduce the effect of spectral leakage and is easy to implement. The classical multichannel sampling theorem and the well-known sampling theorems for the LCT are shown to be special cases of it. Some potential applications of the multichannel sampling are also presented to show the advantage of the theory.

1. Introduction

The linear canonical transform (LCT) appears widely in optics, electromagnetic, and quantum mechanics, as well as in computational and applied mathematics [1–3]. LCT is a class of linear integral transform with three free parameters, which is the general case of many well-known transforms such as the Fourier transform (FT), the fractional Fourier transform (FRFT), the Fresnel transform (FST) and the scaling operations [2–10]. Because concatenation of LCT are also LCT, the family of LCT can model a broad class of optical systems involving arbitrary combinations of any number of lenses, sections of free space, and sections of graded-index media [3]. These systems belong to the class of quadratic-phase systems, which are also known as *ABCD* systems or lossless first-order optical systems [3,9]. The LCT has found many applications in optics, optimal filtering, signal separation, phase retrieval, time-frequency analysis, radar system analysis and many others [3–5,11–22].

Signal reconstruction from its samples is an important signal processing operation, which is needed very frequently in many applications in several diverse areas such as signal processing, communications, geophysics, radar and sonar, and optics including optical signal processing [23,24]. As the LCT has recently been found many applications in signal processing, the sampling theorem expansions for the LCT of compact functions in time domain or LCT domain have been derived from different perspectives [25–36]. The expansions of the classical uniform sampling theorem for the bandlimited or timelimited continuous signal in the LCT domain have been studied [25–31]. These sampling theorems establish the fact that a bandlimited or timelimited signal in the LCT domain can be completely reconstructed by a set of equidistantly spaced signal samples [25–31]. However, there are a variety of applications in which data is sampled with the bandlimited signal through multichannel data acquisition [32–46]. In many practical applications, periodic nonuniform sampling and multichannel sampling schemes for bandlimited signal in the LCT domain are frequently introduced [32–34]. Multichannel sampling for bandlimited signal is fundamental in the theory of multichannel parallel A/D environment and multiplexing wireless communication environment [34–43]. Such as flexible interleaving/multiplexing analog-to-digital (A/D) converter for bandlimited signal [34,41], the orthogonal frequency division multiplexing system based on the LCT for time-frequency selective channels [12], the application in the context of the image superresolution [42–45]. Example in which multichannel sampling may arise in digital flight control, where the velocity as well as the position are recorded [40,41]. As the LCT has found wide applications in optics and signal processing fields, it is theoretically interesting and practically useful to consider the multichannel sampling in the LCT domain.

In this paper, we explore the multichannel sampling from the bandpass signal viewpoint in the LCT domain based on the conventional convolution structure. To the best of our knowledge,

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there are no results published associated with the multichannel sampling for bandpass signal in the LCT domain. It is shown that the proposed multichannel sampling system can be easily implemented with ordinary convolution structure in the time domain. Moreover, the classical Papoulis multichannel sampling in the FT domain [37–39] and the multichannel sampling in the LCT domain [34–36] are shown to be the special cases of the achieved results. This extension of the Papoulis' multichannel sampling expansion in the LCT domain is strongly motivated by its application in the image super-resolution problems where one is required to reconstruct the super-resolution image from the low-resolution LCTed images. Last, some potential applications of the multichannel sampling are presented to show the advantage of the theory. Especially, the proposed multichannel sampling expansion may prove to be useful in the image super-resolution problems and many optical signal processing problems where the distortion can be represented as a simple multiplication by some function in the LCT domain.

The rest of this paper is organized as follows. Section 2 presents the theoretical basis of LCT, bandpass signal and convolution theory. In Section 3, multichannel sampling expansion for bandpass signal in the LCT domain is derived based on conventional convolution structure. Potential applications for multichannel sampling are presented in Section 4. Finally, Section 5 concludes this paper.

2. Preliminaries

2.1. The linear canonical transform

Optical systems involving thin lenses, sections of free space in the Fresnel approximation, sections of quadratic graded-index media, and arbitrary combinations of any number of these are referred to as first-order optical system or quadratic-phase systems. Mathematically, such system can be modeled as LCTs. The output light field $F_A(u)$ a quadratic-phase system is related to its input field $f(t)$ through [3,9].

$$F_A(u) = L_A[f(t)](u) = \begin{cases} \int_{-\infty}^{\infty} f(t)K_A(u,t)\,dt, & b \neq 0, \\ \sqrt{d}e^{j(1/2)cdu^2} f(du), & b = 0, \end{cases} \tag{1}$$

where

$$K_A(u,t) = \sqrt{1/(j2\pi b)}\, e^{j(1/2)[(a/b)t^2 - (2/b)tu + (d/b)u^2]}, \tag{2}$$

where L_A is the unitary LCT operator with parameter matrix $A = (a, b; c, d)$, a, b, c, d are real numbers satisfying $ad - bc = 1$. The transform matrix A is useful in the analysis of optical systems because if several systems are cascaded, the overall system matrix can be found by multiplying the corresponding matrices. The inverse transform for LCT is given by a LCT having parameter $A^{-1} = (d, -b; -c, a)$, that is

$$f(t) = \int_{-\infty}^{\infty} F_A(u)\bar{K}_A(u,t)\,du \tag{3}$$

where the bar denotes complex conjugation.

It should be noted that, when $b = 0$, the LCT of a signal is essentially a chirp multiplication and it is of no particular interest to our objective in this work, so it will not be discussed in this paper. The LCT family includes the FT and FRFT, coordinate scaling, and chirp multiplication and convolution operations as its special cases. When $(a, b;\ c, d) = (\cos\alpha,\ \sin\alpha;\ -\sin\alpha\ \cos\alpha)$, the LCT reduces to the FRFT; when $\alpha = \pi/2$ it reduces to FT with a multiplicative factor. The LCT also reduces to the Fresnel transform if $(a, b; c, d) = (1, b; 0, 1)$. For further details about the definition and properties of LCT [2–4], can be referred.

2.2. The definition of bandpass signals in LCT domain

A signal $f(t)$ is said to be bandpass to Ω_A in the LCT sense with parameter A, if its energy is finite and its LCT $F_A(u)$ vanishes outside the interval $(-u_H, -u_L) \cup (u_L, u_H)$, i.e.,

$$F_A(u) = 0 \quad \text{for} \quad |u| \leq u_H \quad \text{and} \quad |u| \geq u_L \tag{4}$$

$$\int_{-\infty}^{+\infty} \left|f(t)\right|^2 dt = \int_{|u_L| \leq |u| \leq |u_H|} \left|F_A(u)\right|^2 du < \infty \tag{5}$$

where $0 \leq u_L \leq u_H$ and $\Omega_A = u_H - u_L$. Then, a bandpass signal can be reconstructed from its samples by the flowing sampling formula.

$$f(t) = e^{-jat^2/(2b)} \sum_{n=-\infty}^{\infty} f(nT_0) e^{ja(nT_0)^2/(2b)} \left[\frac{\sin\left((t - nT_0)u_H/b\right)}{\left((t - nT_0)u_H/b\right)} - \frac{\sin\left((t - nT_0)u_L/b\right)}{\left((t - nT_0)u_L/b\right)} \right] \tag{6}$$

where the sampling rate $T_0 = \pi b/\Omega_A$. The results can be derived easily by following Li's method [27] associated with the LCT.

2.3. A convolution theorem associated with LCT

A convolution and product structures of the LCT is introduced in [35]

$$f(t) \odot g(t) = e^{-jat^2/(2b)} \left[\left(f(t) e^{jat^2/(2b)} \right) * g(t) \right] \tag{7}$$

where * is the conventional convolution operation for the FT. Then, it is easy to get [35]

$$L_A\left[f(t) \odot g(t)\right](u) = F_A(u) G\left(\frac{u}{b}\right) \tag{8}$$

where $F_A(u)$ and $G(u)$ denotes the LCT of $f(t)$ and the FT of $g(t)$, respectively.

3. Multichannel sampling expansion for bandpass signal in the LCT domain

Multichannel sampling expansion is fundamental in the theory of multichannel parallel A/D environment and multiplexing wireless communication environment. As the LCT has found wide applications in optics and signal processing fields, it is theoretically interesting and practically useful to consider the multichannel sampling expansion based on LCT. In this section, utilizing the conventional convolution structure, the multichannel sampling expansion for bandpass signal with LCT is proposed. The theorem which is constructed by the ordinary convolution in the time domain can reduce the effect of spectral leakage and is easy to implement.

This expansion deals with the configuration shown in Fig. 1. Bandpass signal $f(t)$ is led into M linear systems with system functions

$$h_1(t), h_2(t), \ldots, h_M(t). \tag{9}$$

where $h_k(t)$, $k = 1, \ldots, M$ are conventional bandlimited. We apply to these systems as common input a Ω_A bandpass signal $f(t)$ in the LCT domain. The resulting outputs are M functions:

$$g_k(t) = \bar{B} \int_{-u_H}^{-u_L} F_A(u) H_k\left(\frac{u}{b}\right) e^{-j(a/2b)t^2 + j(tu/b) - j(d/2b)u^2} du + \bar{B} \int_{u_L}^{u_H} F_A(u) H_k\left(\frac{u}{b}\right) e^{-j(a/2b)t^2 + j(tu/b) - j(d/2b)u^2} du, \quad k = 1, 2, \ldots, M \tag{10}$$