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Confinement of electromagnetic radiation in cylindrical symmetry systems

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ABSTRACT

We considered a cylindrical symmetry system where the refractive index ($\mathbf{n}(\rho, \nu)$) varies with the distance from the cylinder axis (ρ). We apply the Fermat's extremal principle in the framework of the geometrical optics to show that radiation traveling through the system could be confined in it. For a given $\mathbf{n}(\rho, \nu)$, a confinement region can be obtained in the $\rho_0\alpha$ plane, ρ_0 and α being two parameters characterizing a given ray. Our simple criterion given by the expression $\rho^2 n^2(\rho) = [n^2(\rho_0)\sin^2\alpha]\rho^2 + [\rho_0^2 n^2(\rho_0)\cos^2\alpha]$ may be used to improve the design of optical devices.

1. Introduction

Theory of light rays propagating in media with varying refractive indices is of great interest in many fields of theoretical and applied physics; from the study of radiative transfer in planetary and stellar atmospheres to the development of integrated optics, lenses and waveguides, widely used in communication systems. Regarding the last point, the self-focusing optical waveguides have been attracting growing interest because they are suitable for data transmission with low attenuation in broadband systems. The most successful efforts to create this type of waveguide was made by Uchida et al. [1].

An extensive literature concerning the confinement and guiding of radiation in media with varying refractive index, usually called GRIN media, does exists. As explained in [2], “a GRIN lens consists of a cylinder of inhomogeneous dielectric material with a refractive index distribution that has a maximum at the cylinder axis and decreases continuously from the axis to the periphery along the transverse direction”. The refractive index of a GRIN lens can be mathematically represented according to the technique used to make it. For example, GRIN lenses made of a radial gradient-index material obtained by ion exchange, such as SELFOC(r) lens, are characterized by a refractive index that varies according to a parabolic function. In the most general case, the refractive indices of the media can be represented by a hyperbolic secant function of some spatial coordinate. Actually, the parabolic function can be considered as a first order approximation of Taylor series of hyperbolic secant function [3,4].

In this paper we are interested on guided waves through inhomogeneous media into an infinite cylindrical region where the refractive index depends on the radial coordinate, so that it includes the models we have mentioned above. Firstly, we aim at showing that, for given initial conditions and for a specific refractive index that varies with the radial coordinate in cylindrical coordinates, the incident radiation would remain confined indefinitely within the region, without escaping. Secondly, we are interested on finding a simple criterion to realize that radiation capture is occurring within a given waveguide with diffuse edges.

In a previous article [5], we have analyzed the captured radiation in a sphere with diffuse edges where the refractive index varies smoothly with the distance to the center of the sphere. On that occasion we have found a simple criterion to realize that radiation capture is occurring in a given system. In the present work, we analyze the confinement of radiation in a system with cylindrical symmetry and diffuse edges, where the refractive index, $\mathbf{n}(\rho, \nu)$, varies smoothly with the distance to the cylinder axis, ρ. Therefore, the procedure we follow to develop our ideas is the same as that we have explained in [5]. More specifically, we apply the Fermat's extremal principle in the framework of the geometrical optics to analize the confinement of the guided radiation in a given

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domain. Ray tracing in GRIN media has been extensively studied in order to implement many optical devices. Analytical solutions of the ray tracing problem can be found in [6–8]. An interesting work is the paper by Evans [9]. By comparing the Newtonian mechanics with the laws defining geometrical optics, the author has calculated the three-dimensional trajectory of a ray propagating through a medium with a parabolic refractive index.

As noted above, in this paper we have worked just as we did it in [5]. However, we must point that there are two important differences: (1) in this paper the trajectories are twisted curves in three-dimensional space, instead of plane curves like the ones we studied in [5], and (2) in the present paper we have addressed waveguides and we have obtained guided radiation, instead of captured radiation like in the previous article.

Moreover, we have found a quite simple criterion, which depends on two parameters characterizing a given ray (ρ_0 and α), that allows us to determine whether the ray will be confined or not in a given system.

This paper is organized as follows: in Section (2) we show the condition of total internal reflexion for an homogeneous transparent cylinder, which is characterized by a refractive index $\mathbf{n}>1$ relative to the surrounding medium. In Section (3), we extend the analysis to systems where $\mathbf{n}(\rho, \nu)$ varies smoothly with ρ, the distance to the cylinder axis. In Section (4), we present our conclusions.

2. A simplified model. Systems with a step at the refractive index

Let us consider an homogeneous transparent cylinder of radius R and infinite length, which is characterized by a refractive index $\mathbf{n}$ >1 relative to the surrounding medium (just to simplify, in this and the following Sects. we get rid of the $\mathbf{n}$ dependence on ν). If electromagnetic radiation inside the cylinder responds to geometrical optics laws, a description based on the light ray concept is valid. Let us consider a light ray propagating in an arbitrary direction within the cylinder. If the ray is not traveling parallel to the cylinder axis, it eventually strikes the cylinder wall, in which it is reflected and refracted. The reflected light continues traveling inside the cylinder until it again strikes the cylinder wall where part of the light is reflected. The process is repeated over and over again. Let us consider a portion of this endless journey into the cylinder between two successive reflections in the cylinder wall. Being P_1 and P_2 the points where the ray strikes the cylinder wall, we define a Cartesian coordinate system so that the Z axis is along the cylinder axis, the ray is on a plane which is parallel to the Y–Z plane, and the X–Y plane intersects the ray at P_0, the point which is equidistant from P_1 and P_2 (see Fig. 1). Thus, the X coordinate of P_0 is the minimum distance of the ray from the origin of the coordinate system.

To represent the position vectors of the points P_1 and P_2 with respect to the coordinate system that we have just stated, we can write

$$\vec{r}_1 = (x, -y, -z), \quad \vec{r}_2 = (x, y, z). \tag{1}$$

Therefore,

$$\vec{r}_2 - \vec{r}_1 = (0, 2y, 2z), \quad \vec{r}_1 - \vec{r}_2 = (0, -2y, -2z) \tag{2}$$

The internal normal vectors to the cylinder wall at the points P_1 and P_2 can also be written in terms of its coordinates as

$$\vec{n}_1 = (-x, y, 0), \quad \vec{n}_2 = (-x, -y, 0). \tag{3}$$

From the dot products between the vectors $\vec{n}_1$ and $(\vec{r}_2 - \vec{r}_1)$, and between the vectors $\vec{n}_2$ and $(\vec{r}_1 - \vec{r}_2)$, the angles are θ_1 and

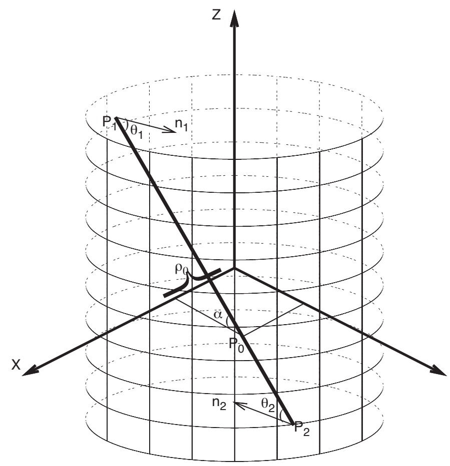

Fig. 1. Used coordinate system (see text for definition of the parameters).

θ_2, respectively, between the ray of light and the internal normal to the cylinder wall at the points P_1 and P_2 are found:

$$\cos\theta_1 = \frac{\vec{n}_1 \cdot \left(\vec{r}_2 - \vec{r}_1\right)}{|\vec{n}_1|\,|\vec{r}_2 - \vec{r}_1|} = \frac{2y^2}{(x^2+y^2)^{1/2}(4y^2+4z^2)^{1/2}} \tag{4}$$

$$\cos\theta_2 = \frac{\vec{n}_2 \cdot (\vec{r}_1 - \vec{r}_2)}{|\vec{n}_2|\,|\vec{r}_1 - \vec{r}_2|} = \frac{2y^2}{(x^2+y^2)^{1/2}(4y^2+4z^2)^{1/2}}. \tag{5}$$

As follows from the last expressions, $\theta_1 = \theta_2$. This means that a given ray propagating into the cylinder strikes the wall always with the same angle.

From the Snell law, we can ascertain the condition of total internal reflexion as

$$\sin\theta > \frac{1}{n}. \tag{6}$$

Therefore,

$$\cos^2\theta = 1 - \frac{1}{n^2} \tag{7}$$

represents the limit condition. From Eqs. (4) or (5) we can write for the limit condition

$$\frac{y^4}{(x^2+y^2)(y^2+z^2)} = 1 - \frac{1}{n^2}. \tag{8}$$

From the last expression, and taking into account that $x^2 + y^2 = R^2$, we obtain

$$z^2 = \left[\frac{n^2}{R^2\left(n^2-1\right)}\right] x^4 - \left[\frac{n^2+1}{n^2-1}\right] x^2 + \left[\frac{R^2}{n^2-1}\right] \tag{9}$$

or

$$z^2 = a(x^4 + bx^2 + c), \tag{10}$$

being

$$a = \frac{n^2}{R^2(n^2-1)}, \quad b = \frac{R^2(n^2+1)}{n^2}, \quad c = \frac{R^4}{n^2}. \tag{11}$$

Therefore, a, b, and c are constants for a given system.

Solving Eq. (10) we obtain

$$z^2 = a(x^2 - x_1^2)(x^2 - x_2^2) \tag{12}$$

with

$$x_1 = \frac{R}{n}, \quad x_2 = R \tag{13}$$