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Sine hollow beam and its propagation property

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ABSTRACT

The normal sine hollow beam (NSHB) and anomalous sine hollow beam (ASHB) are proposed to describe the dark hollow beam (DHB). Based on the Collins formula, an analytical formula for NSHB (ASHB) through the ABCD optical system is derived. The intensity distributions of NSHB (ASHB) are characterized by the beam parameters and the propagation size. As the numerical example, the propagation properties of NSHB (ASHB) through the ABCD optical system have been demonstrated graphically. It is shown that NSHB (ASHB) will be evolved to the solid beam having the maximum light intensity in the beam center in free space. However, it will make the laser energy concentrate in the small area for NSHB (ASHB) through the convergent optical system.

1. Introduction

Dark hollow beam (DHB) with zero central intensity has attracted much attention due to its increasing application in atom optics [1,2]. There are many methods for creating the DHB, such as Laguerre Gaussian beam [3], high-order Bessel beam [4], coherent combination of array beams [5], annular beam generated by annular laser [6] and so on. Recently, Wu observed experimentally an anomalous hollow beam, which can be used for studying the linear and nonlinear particle dynamics in the storage ring [7]. The propagation properties for anomalous hollow beam through the optics system have been studied by [8–10], such as paraxial optics system [8], the turbulent atmosphere [9], misaligned optics system [10] and so on. In this paper, the sine hollow beam (SHB) is described by the sine function, and its propagation properties through ABCD optical system are studied in detail. It is different from the former work [11,12], in this paper, a new analytical expression for propagation of NSHB (ASHB) is given by trigonometric function through using the identity of trigonometric function. The propagation properties of NSHB (ASHB) through ABCD optical system are illustrated graphically. However, the propagation expression for SHB through ABCD optical system is given by the superposition of a series of hypergeometric function and Gamma function [11]. The propagation expression [12] is determined by the a few Gaussian functions. Therefore, it has important practical significance to the further study of NSHB (ASHB) through complex optical system.

2. SHB and propagation formula

We define the electric field of the NSHB (ASHB) at the original plane of $z = 0$ as follows:

$$E_0(x_0, y_0, 0) = G_0\left\{\sin\left[\frac{x_0^2}{a_0 w_0^2} + \frac{y_0^2}{b_0 w_0^2}\right]\right\}^n \times \exp\left[-\frac{x_0^2 + y_0^2}{w_0^2}\right]. \quad (1)$$

where, $G_0 = 1$ is the distribution constant, $w_0 = 3$ mm is size of beam waist, two constants a_0 and b_0 are related to the intensity distribution, n is the order of SHB. Eq. (1) denotes the light field of NSHB for a_0 equals to b_0. Conversely, it is ASHB. Setting the wavelength $\lambda = 10.6\ \mu$m, and the normalized intensity distributions of ASHB at the original plane for the different parameters n are shown in Fig. 1(a). The peak light intensity (PLI) of ASHB will drop and the dark areas will become wide with increase of low order n, however, the changes are unconspicuous when values of n are more than 5. The normalized intensity distributions of NSHB (ASHB) for the same parameter $n = 3$ are shown in Fig. 1(b). It is obvious that PLI of NSHB is much more than PLI of ASHB, however, the widths of dark areas for NSHB and ASHB keep invariable approximately with increase of parameters a_0 and b_0. It is a unique model for studying the transverse instabilities, in particular, the interaction of the wake-field and the lattice nonlinearity.

The propagation of any beam through ABCD optical system can be given by the Collins formula, therefore, the light field in the observation plane is given by

$$E(x, y, z) = \frac{ik\exp(-ikz)}{2\pi B}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} E_0(x_0, y_0, 0)$$

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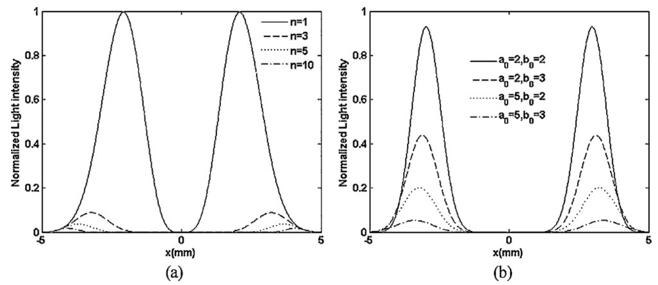

Fig. 1. Diagrams of the normalized light intensity distributions of NSHB (ASHB) for various a_0, b_0 and n, (a) $a_0 = 2$, $b_0 = 5$ and (b) $n = 3$.

$$\times \exp\left\{-\frac{ik}{2B}[A(x_0^2+y_0^2)-2(xx_0+yy_0)+D(x^2+y^2)]\right\}$$
$$\times\, dx_0 dy_0. \tag{2}$$

Using the following integral formula

$$\int_{-\infty}^{+\infty} \exp[-(ax^2+2bx+c)]\cdot\sin(px^2+2qx+r)dx$$
$$= \frac{\sqrt{\pi}}{(a^2+p^2)^{1/4}} \times \exp\left[\frac{a(b^2-ac)-(aq^2-2bpq+cp^2)}{a^2+p^2}\right]$$
$$\cdot \sin\left[\frac{1}{2}\arctan\frac{p}{a}-\frac{p(q^2-pr)-(b^2p-2abq+a^2r)}{a^2+p^2}\right]. \tag{3}$$

$$\int_{-\infty}^{+\infty} \exp[-(ax^2+2bx+c)]\times\cos(px^2+2qx+r)dx$$
$$= \frac{\sqrt{\pi}}{(a^2+p^2)^{1/4}} \times \exp\left[\frac{a(b^2-ac)-(aq^2-2bpq+cp^2)}{a^2+p^2}\right]$$
$$\cdot \cos\left[\frac{1}{2}\arctan\frac{p}{a}-\frac{p(q^2-pr)-(b^2p-2abq+a^2r)}{a^2+p^2}\right]. \tag{4}$$

Substituting Eq. (1) into Eq. (2) and setting $n = 1$, an analytical expression for NSHB (ASHB) through ABCD optical system is given by

$$E(x,y,z) = \frac{ik\exp(-ikz)}{2B}\cdot[(a_c^2+p_1^2)(a_c^2+p_2^2)]^{-1/4}$$
$$\times \exp\left[-\frac{ikD}{2B}(x^2+y^2)\right]\exp\left(\frac{a_c b_1^2}{a_c^2+p_1^2}+\frac{a_c b_2^2}{a_c^2+p_2^2}\right)$$
$$\cdot \sin(\xi+\sigma). \tag{5}$$

where

$$a_c = \frac{ikAw_0^2+2B}{2Bw_0^2}. \tag{6}$$

$$b_1 = -\frac{ikx}{2B}. \tag{7}$$

$$b_2 = -\frac{iky}{2B}. \tag{8}$$

$$p_1 = (a_0 w_0^2)^{-1}. \tag{9}$$

$$p_2 = (b_0 w_0^2)^{-1}. \tag{10}$$

$$\xi = \frac{1}{2}\arctan\frac{p_1}{a_c}+\frac{b_1^2 p_1}{a_c^2+p_1^2}. \tag{11}$$

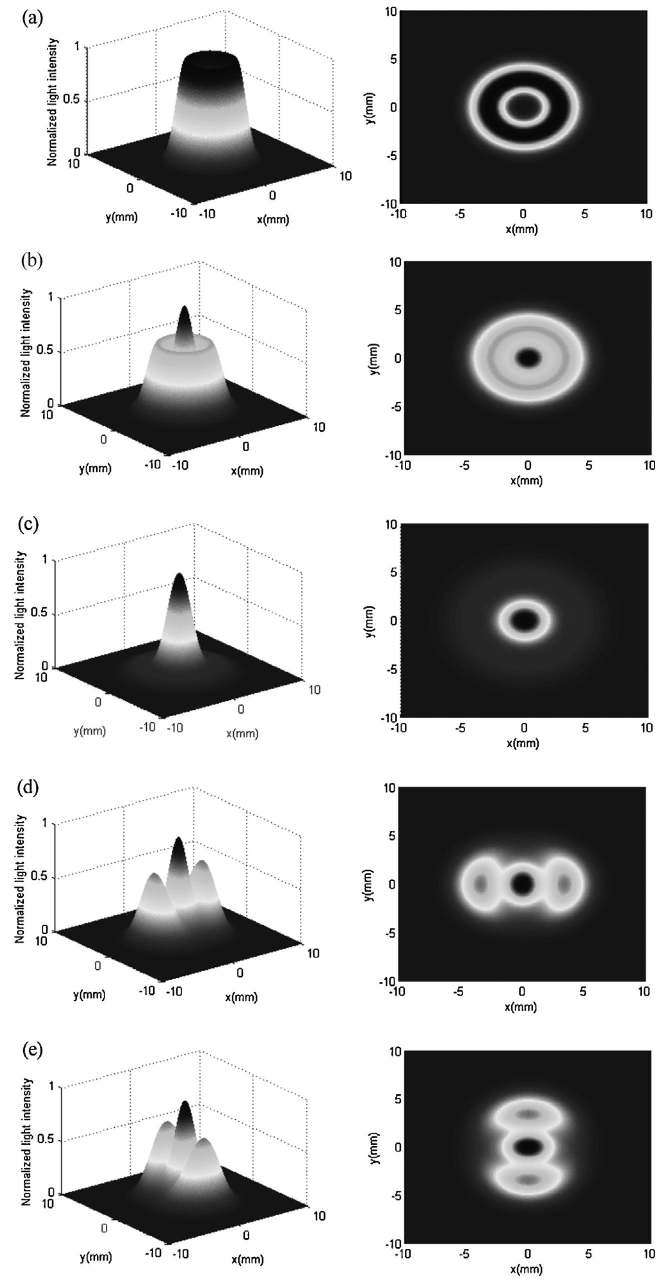

Fig. 2. Diagrams of the normalized light intensity distributions of NSHB (ASHB) and top views, (a) $a_0 = 2$, $b_0 = 2$, $z = 0.1$ m; (b) $a_0 = 2$, $b_0 = 2$, $z = 1.1$ m; (c) $a_0 = 2$, $b_0 = 2$, $z = 2.5$ m; (d) $a_0 = 2$, $b_0 = 5$, $z = 1.5$ m; and (e) $a_0 = 5$, $b_0 = 2$, $z = 1.5$ m.

$$\sigma = \frac{1}{2}\arctan\frac{p_2}{a_c}+\frac{b_2^2 p_2}{a_c^2+p_2^2}. \tag{12}$$

where, A, B, C and D are propagation matrix elements, the light intensity $I(x, y, z) = [E(x, y, z)][E(x, y, z)]^*$.

3. Numerical examples

3.1. Free space

As an example, we will study the light intensity for NSHB (ASHB) through the free space, therefore, the propagation matrix is given by

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & z \\ 0 & 1 \end{pmatrix}. \tag{13}$$