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Generating chaos via nonlinear system switching anti-control and circuit implementation

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ABSTRACT

In this paper, a novel approach is developed for generating chaos based on an unstable nonlinear system switching anti-control strategy and constructing heteroclinic loops, which is different from the existing linear system switching-based chaotification. To confirm the existence of chaos, a topological horseshoe of the proposed switching controlled nonlinear system is further investigated. In addition, a circuit is also designed and implemented, with experimental results demonstrated. Both numerical simulations and circuit implementation together show the effectiveness of the proposed systematic methodology.

1. Introduction

Chaos control refers to purposefully eliminating or weakening chaotic behavior of systems through control methods when the chaotic motion is harmful. Since the OGY method was proposed in 1990 [1], much effort has been devoted to the study of controlling chaos. However, not all chaotic behaviors are harmful, and recent research has shown that the distinct properties of chaos, such as positive Lyapunov exponents, topological transitivity, quasi-randomness, sensitively dependence on initial conditions and system parameters, can actually be useful under certain circumstances, in such as liquid mixing, information processing, flexible systems design and secret communications. Therefore, chaotification by means of making an originally non-chaotic dynamical system chaotic, or enhancing existing chaos, has attracted some special attention recently [2–5].

In many existing switching chaotic systems, it is a common practice that chaos is generated by only using linear system switching-based strategy. For example, Li et al. presents an approach for constructing piecewise linear chaotic system by selecting one linear system as switching anti-control [6]. Yu et al. use two linear systems for constructing piecewise switching chaotic systems, according to heteroclinic Shil'nikov theorem [7,8]. One may ask whether or not there is a more generalized way further to break such a limitation so as to generate chaos by means of nonlinear system switching anti-control? This paper gives a positive answer to the question.

In this paper, different from the existing linear system switching-based chaos generation, a more generalized methodology is developed here for chaotification by means of making an originally unstable nonlinear dynamical system chaotic via switching anti-control strategy. The main differences between the linear and nonlinear system switching anti-control for chaos generation lie in: (i) since linear systems are special cases of nonlinear systems, the presented new method is also applicable to linear system switching anti-control in general; (ii) in phase space, the phase diagrams of chaotic attractors generated by linear system switching anti-control are symmetric with respect to the vertical axis through the origin, while that of chaotic attractors generated by nonlinear system switching anti-control are not symmetric with respect to the vertical axis through the origin. Furthermore, by picking a suitable cross-section with respect to the proposed switching-based nonlinear system carefully, a topological horseshoe of the corresponding first-returned Poincare map can be found, confirming the existence of chaos in the proposed switching-based nonlinear system. Finally, a circuit is also designed and implemented, with experimental results demonstrated. Both numerical simulations and circuit implementation together show the effectiveness of the proposed systematic methodology.

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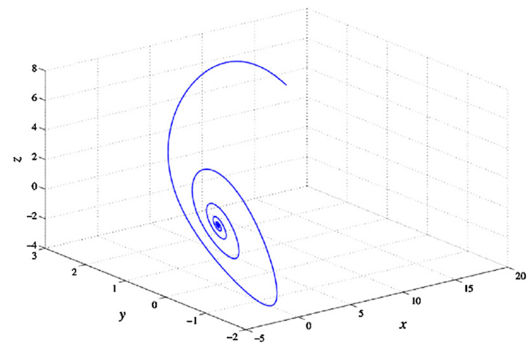

Fig. 1. The phase diagram of unstable nonlinear system (1).

The rest of the paper is organized as follows. A basic unstable nonlinear system is given in Section 2. Generating chaos via nonlinear system switching anti-control is proposed in Section 3. A topological horseshoe of the proposed switching-based nonlinear system is further investigated in Section 4. A circuit is designed and implemented in Section 5. Finally, Section 6 concludes the paper.

2. A basic unstable nonlinear system

Consider a basic three-dimensional nonlinear system:

$$\begin{cases} \dot{x} = az \\ \dot{y} = by + z \\ \dot{z} = cx + y + y^2 \end{cases} \tag{1}$$

where $a = 1.6$, $b = -2$, $c = -1$ are parameters. Obviously, the nonlinear system (1) is unstable, with a unique equilibrium at origin $O(0, 0, 0)$, as shown in Fig. 1.

Linearizing system (1) at $O(0, 0, 0)$, one gets the Jacobian matrix as follows:

$$J = \begin{pmatrix} 0 & 0 & a \\ 0 & b & 1 \\ c & 1 & 0 \end{pmatrix} \tag{2}$$

According to (2), the corresponding eigenvalues are $\gamma = -2.3314$, $\sigma \pm j\omega = 0.1657 \pm j1.1598$. Thus, $O(0, 0, 0)$ is the saddle foci with index 2. Moreover, their corresponding eigenvectors are given as follows:

$$\begin{cases} \mu = \xi_\gamma = \begin{pmatrix} 0.2110 \\ 0.9279 \\ -0.3075 \end{pmatrix} \\ \eta = \xi_R \pm j\xi_I = \begin{pmatrix} 0.7845 \\ 0.1384 \\ 0.0812 \end{pmatrix} \pm j \begin{pmatrix} 0 \\ 0.1884 \\ 0.5686 \end{pmatrix} \end{cases} \tag{3}$$

In (3), the one dimensional stable manifold $E^S(O)$ corresponding to the real eigenvalue $\gamma = -2.3314$ and the two dimensional unstable manifold $E^U(O)$ corresponding to the complex conjugate eigenvalues $\sigma \pm j\omega = 0.1657 \pm j1.1598$ at O are given by:

$$\begin{cases} E^S(O) : \dfrac{x}{l} = \dfrac{y}{m} = \dfrac{z}{n} \\ E^U(O) : Ax + By + Cz = 0 \end{cases} \tag{4}$$

where (l, m, n) are the direction vector of one dimensional stable manifold $E^S(O)$ with $l = 0.2110$, $m = 0.9279$, $n = -0.3075$, and (A, B, C) are the direction vector of two dimensional unstable manifold $E^U(O)$ with $A = 0.0634$, $B = -0.4461$, $C = 0.1478$.

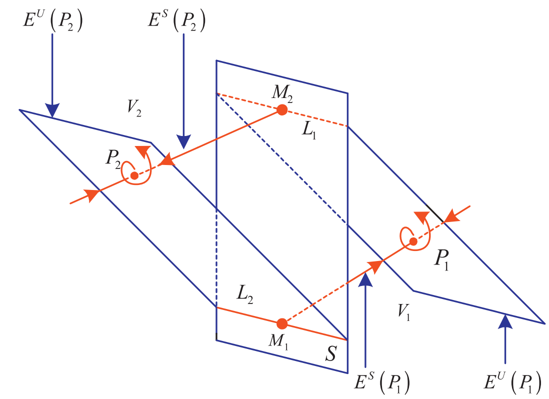

Fig. 2. The heteroclinic loop in the eigenspace of system (5).

3. Generating chaos via nonlinear system switching anti-control

For classical control theory, assume that the uncontrolled nonlinear system is unstable, by designing a switching controller, such that the controlled system achieves stabilization or robust stability. In contrast with classical control method, in this section, a generalized approach is developed for chaotification by means of making an originally unstable nonlinear dynamical system (1) chaotic, by using switching anti-control strategy.

Let the switching plane be $S = \{(x, y, z)|y = 0\}$. From (1), one can construct the switching system, given by:

$$\begin{cases} \dot{x} = a(z - f_3(x, y, z)) \\ \dot{y} = b(y - f_2(x, y, z)) + (z - f_3(x, y, z)) \\ \dot{z} = c(x - f_1(x, y, z)) + (y - f_2(x, y, z)) + (y - f_2(x, y, z))^2 \end{cases} \tag{5}$$

where f_1, f_2 and f_3 are switching controller, and its detail mathematical expression is determined by the existence conditions of heteroclinic loop in system (5).

The switching plane divided the state space into two subspaces: V_1 and V_2, and in each subspace f_1, f_2 and f_3 are switching functions. Clearly, the system (5) is 2-piecewise nonlinear system. System (5) has two equilibrium points $P_1(x_1, y_1, z_1) \in V_1$ and $P_2(x_2, y_2, z_2) \in V_2$, they are located at opposite sides of the switching plane $S = \{(x, y, z)|y = 0\}$. Because the Jaconbian matrix of system (5) at P_1 and P_2 is the same with (2), so the eigenvalues and the eigenvectors of system (5) at P_1 and P_2 is the same with (3). At the same time, the direction vector of one dimensional stable manifold $E^S(O)$ and the direction vector of two dimensional unstable manifold $E^U(O)$ is the same with (4).

Based on the above analysis, the one dimensional stable manifold $E^S(P_1)$ and the two dimensional unstable manifold $E^U(P_1)$ of system (5) at equilibrium points $P_1(x_1, y_1, z_1) \in V_1$ are described by:

$$\begin{cases} E^S(P_1) : \dfrac{x - x_1}{l} = \dfrac{y - y_1}{m} = \dfrac{z - z_1}{n} \\ E^U(P_1) : A(x - x_1) + B(y - y_1) + C(z - z_1) = 0 \end{cases} \tag{6}$$

Similarly, the one dimensional stable manifold $E^S(P_2)$ and the two dimensional unstable manifold $E^U(P_2)$ of system (5) at equilibrium points $P_2(x_2, y_2, z_2) \in V_2$ are given by:

$$\begin{cases} E^S(P_2) : \dfrac{x - x_2}{l} = \dfrac{y - y_2}{m} = \dfrac{z - z_2}{n} \\ E^U(P_2) : A(x - x_2) + B(y - y_2) + C(z - z_2) = 0 \end{cases} \tag{7}$$

Next, according to the heteroclinic Shil'nikov theorem [9], one can seek the conditions that the coordinates of equilibrium points