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Soliton switching study using new normalized nonlinear Schrödinger equations

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ABSTRACT

Soliton switching in a nonlinear directional coupler (NLDC) is studied by using new normalized nonlinear Schrödinger equations (NLSE). The resulted equations have one parameter κL_D only, this feature greatly simplifies the numerical study of soliton coupling in a NLDC, and it enables us to derive the soliton switching conditions based on numerical analysis. Numerical results show that most of the input soliton energy remains in the launched waveguide when input soliton normalized peak power is bigger than 2.5. It is contrary to the existing theoretical expectations which were derived using variational approach. The fundamental soliton switching conditions is also expressed in terms of input soliton pulse width. In addition, it is found that frequency chirp degrades soliton switching performance in a NLDC.

1. Introduction

Optical switching in a nonlinear directional coupler (NLDC) was first studied in 1982. The nonlinear coupled equations, which describe the continuous wave (cw) optical coupling in a NLDC, and their analytical solutions were derived [1,2]. Theoretical and experimental results showed that the quasi-cw optical pulse breaks up at the output of the NLDC according to its instantaneous power. This kind of pulse breakup seriously degrades the performance of NLDC optical switch [3]. To improve the NLDC switching performance, soliton switching was proposed, and the nonlinear Schrödinger equations (NLSEs) were rewritten in soliton normalization to study soliton coupling in a NLDC [4]. By then, there were two sets of equations describing optical switching in a NLDC. One set of equations describe cw and quasi-cw switching and another one set of equations describe soliton switching. Theoretical results shown that a soliton switching shows a much better switching performance compared with a quasi-cw optical switching. The question then arises: what kinds of optical pulse can be regarded as a quasi-cw in a NLDC? Can one set of equations describe cw coupling, pulse coupling and also soliton coupling in a NLDC? To answer these questions, we introduced the new normalized nonlinear Schrödinger equations (NLSEs) to study coupling in a NLDC [5–7]. In the new normalized NLSEs, z is normalized to coupling coefficient but not dispersion length. As a result, $z = \pi/2$ corresponds to a half beat length of coupler. Amplitudes of the optical pulses are normalized to the critical power. It is straightforward to tell whether the NLDC is operating in the nonlinear coupling regime or in the linear regime where the influence of nonlinearity on the coupling behaviors is negligible. The nonlinearity in a NLDC is negligible and NLDC reduces to linear coupler if the input peak power is less than half of the critical power [8].

By using this new normalized NLSEs, we have found that coupling behaviors in a NLDC mainly depends on a dimensionless parameter κL_D. When $\kappa L_D \gg 1$, the dispersive effects play a minor role, pulse coupling is reduced to quasi-cw coupling and follow cw coupling equations. Both the pulse coupling and cw coupling in an NLDC have the similar energy transfer characteristics [5–7,9]. This theoretical prediction agrees well with experimental results of a NLDC femoto-second switching [10]. The experiments shown that energy transfer characteristics of a 100 fs pulse in a NLDC is very close to that of a continuous wave. This is because its $\kappa L_D \approx 58 \gg 1$ according to Ref. [6]. When $\kappa L_D < 1$, the dispersive effects become important, the NLDC operating in the anomalous dispersion regime exhibits good switching performance. Obviously, in this new normalization NLSEs, cw coupling, quasi-cw coupling and pulse coupling are related and they are described by one set of

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equations. When the normalized peak power is less than 0.5, nonlinear coupling is reduced to linear coupling. In this paper, we will show that soliton switching in a NLDC can also be described by the new normalized NLSEs.

The Soliton switching has been studied by using both numerical method and variational approach in the soliton normalized NLSEs [4,9,11–14]. Although there is one parameter in the soliton normalized NLSEs, the normalized power, normalized length and the normalized coupling coefficient are the functions of the dispersion length. All these parameters are related implicitly, as a result, the numerical analysis of the soliton coupling in a NLDC was not simplified effectively. The variational approach, precisely Kantorovich method which has been extensively used to solve partial differential equations in mechanics, was first introduced to study the NLDC by Paré and Florjańczyk [11]. It is well known that choosing appropriate trial functions is crucial when Kantorovich variational approach is used to solve partial differential equations. Generally, the more parameters are in the trail functions, the closer the results are to the exact solutions [11,14]. There is a tradeoff between number of parameters in the trail functions and accuracy of the solutions. By assuming the trail functions to be a fundamental soliton $\eta_1 \operatorname{sech}(\eta_1 \tau)$, Kivshar studied soliton switching in a NLDC and derived the soliton switching conditions analytically. Soliton self-switching occurs in a half-beat length NLDC only when the normalized peak power satisfies $\pi/2 < P_i/P_c < 2.43$, since most of the soliton energy remains in the core in which the soliton is launched initially [9,13]. It is contrary to our results in this paper, soliton self switching happens in a half-beat length NLDC when $P_i/P_c \geq 2.5$. Realized the limitations of fundamental soliton as trail functions, Kivshar and his coworkers introduced trial functions with more parameters studying soliton switching in their subsequent paper [14]. Other trial solutions were also introduced to variational analysis of soliton switching to get the approximating solution closer to the exact solution [9,12]. However, with more variables in the trial solutions, analytical solutions because impossible in the variational analysis and numerical analysis become necessary. As a result, it loses the advantages of variational analysis, and can't give the soliton switching conditions.

In this paper, we will study the soliton switching by using the new normalization. In the new normalization, the input soliton shall be expressed as $(1/4\kappa L_D)^{1/2} \operatorname{sech}(\tau)$ to ensure the nonlinear length equals to the dispersion length. Obviously, when $\kappa L_D \geq 1$, the normalized peak power of the input soliton is less than 0.25, hence the nonlinearity is negligible and nonlinear coupling reduces to linear coupling. Soliton can couple back and forth in a coupler. As a result, when $\kappa L_D \geq 1$, soliton switching is operating in the linear regime, and the coupling behaviors of a soliton with $\kappa L_D \geq 1$ are same as $\kappa L_D \geq 1$. So the simulation range of κL_D is reduced to [0 1] from [0 ∞]. There is one parameter only in the new normalization soliton switching system. By doing a nested for-loop simulation of parameter κL_D and coupling length z, we can get all the information of a fundamental soliton switching characteristics in a NLDC. The outer for-loop variable κL_D is from 0 to 1, and the inner for-loop variable z is from 0 to $\pi/2$. In a half-beat length linear coupler, the launched optical energy completely transfers to the adjacent waveguide from the launched waveguide; while in a half beat length NLDC, most of the launched optical energy remains in the launched waveguide initially. The energy transferring efficiency between two coupled waveguides in a half beat length NLDC as a function of κL_D can be easily obtained by simulating once. These features of the new normalized NLSEs enable us to understand the soliton coupling behaviors and to derive the soliton switching conditions based on a very few of numerical simulations. Simulation results show that Soliton self-switching occurs in a NLDC when $P_i/P_c \geq 2.5$, that is, most of the input soliton energy stays in the launched waveguide.

2. Theory

The evaluation of pulses in a NLDC could be described by a coupled extended nonlinear Schrödinger equations [7,9,15],

$$\frac{\partial A_j}{\partial z} = -\beta_1 \frac{\partial A_j}{\partial t} - i\frac{\beta_2}{2}\frac{\partial^2 A_j}{\partial t^2} + \frac{\beta_3}{6}\frac{\partial^3 A_j}{\partial t^3} - \frac{\alpha}{2}A_j + i\kappa A_{3-j} - \eta\frac{\partial A_{3-j}}{\partial t} + i\gamma\left(1 + \frac{i}{\omega_0}\frac{\partial}{\partial t}\right)\left(A_j\int_{-\infty}^{\infty} R\left(t'\right)\left|A_j\left(t-t'\right)\right|^2 \mathrm{d}t'\right) \tag{1a}$$

where $A_j(z,t)$, $j = 1,2$, is the slowly varying envelope amplitude of the modal field in waveguide j. β_1, β_2 and β_3 are, respectively, the 1st order, the 2nd order and 3rd order dispersion. γ and α are, respectively, the nonlinear and loss or gain coefficients in a waveguide [16]. η is the intermodal dispersion coefficient. κ is linear coupling coefficient. ω_0 is the center frequency. $R(t)$ includes the instantaneous nonlinear response and the retarded Raman response, and it could be written as [9,17],

$$R(t) = (1 - f_R)\delta(t) + f_R h_R(t) \tag{1b}$$

f_R is a constant and $f_R = 0.18$ in silica fibers. $h_R(t)$ is the Raman response function. We employed Blow and Wood's approximate form, $h_R(t) = 0.09388 \exp(-t/32)\sin(t/12.2)$, the unit of time is fs [9].

Here we analyze Eq. (1) by adopting a new normalization, which facilitates the study and understanding of the coupling dynamics in a NLDC [5–7]. By taking the transformation of $\tau = (t - \beta_1 z)/T_0$, $Z = z\kappa$, $a_j = A_j/\sqrt{P_c}$ and $P_c = 4k/\gamma$, Eq. (1) can be expressed as,

$$\frac{\partial a_j}{\partial Z} = -i\frac{\operatorname{sgn}(\beta_2)}{2L_D\kappa}\frac{\partial^2 a_j}{\partial \tau^2} + \frac{\operatorname{sgn}(\beta_3)}{6L_D'\kappa}\frac{\partial^3 a_j}{\partial \tau^3} - \frac{\alpha}{2\kappa}a_j + ia_{3-j} - \mathrm{IMD}\frac{\partial a_{3-j}}{\partial \tau} + i4(1-f_R)\left|a_j\right|^2 a_j + i4f_R a_j\int_0^{\infty} h_R(\tau')\left|a_j(\tau-\tau')\right|^2 \mathrm{d}\tau' - \frac{4(1-f_R)}{\omega_0 T_0}\frac{\partial\left(a_j\left|a_j\right|^2\right)}{\partial \tau} - \frac{4f_R}{\omega_0 T_0}\frac{\partial\left(a_j\int_0^{\infty} h_R(\tau')\left|a_j(\tau-\tau')\right|^2 \mathrm{d}\tau'\right)}{\partial \tau} \tag{2}$$

where $h_R(\tau') = 0.09388T_0 \exp(-\tau' T_0/32)\sin(\tau' T_0/12.2)$, T_0 is the input pulse width. $\mathrm{IMD} = \eta/(\kappa T_0)$, $L_D = T_0^2/\left|\beta_2\right|$ is the 2-order dispersion length, $L_D' = T_0^3/\left|\beta_3\right|$ is 3-order dispersion length. Z is the normalized length of the NLDC. Eq. (2) describes the coupling behaviors of both pulses and cw in a NLDC, it can be numerically solved [7]. If the input pulse is much long compared with the Raman response time and $1/\omega_0 T_0 \ll 1$, both self-steepening effect and retarded nonlinear response can be ignored and Eq. (2) reduces to,

$$\frac{\partial a_j}{\partial Z} = -i\frac{\operatorname{sgn}(\beta_2)}{2L_D\kappa}\frac{\partial^2 a_j}{\partial \tau^2} + \frac{\operatorname{sgn}(\beta_3)}{6L_D'\kappa}\frac{\partial^3 a_j}{\partial \tau^3} - \frac{\alpha}{2\kappa}a_j + ia_{3-j} - \mathrm{IMD}\frac{\partial a_{3-j}}{\partial \tau} + i4\left|a_j\right|^2 a_j \tag{3}$$

By ignoring the high order dispersion terms and loss term, Eq. (3) becomes,

$$\frac{\partial a_j}{\partial Z} = i\frac{1}{2\kappa L_D}\frac{\partial^2 a_j}{\partial \tau^2} + i4\left|a_j\right|^2 a_j + ia_{3-j} \tag{4}$$

Eq. (4) can be used to study pulse coupling in a NLDC, it can also be used to study cw coupling and linear coupling. When $kL_D \gg 1$, dispersion term in Eq. (4) can be ignored and Eq. (4) reduces to the cw nonlinear coupled equations. When the input pulse peak power is much less than 1, the nonlinear term in Eq. (4) can be ignored and