Optik 125 (2014) 3759–3763

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Study on compressed sensing imaging based on intensity modulation in Fourier domain

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ARTICLE INFO

Article history:
Received 20 July 2013
Accepted 15 January 2014

Keywords:
Compressed sensing
Intensity modulation
Coherence

ABSTRACT

In 4F system, compressed sensing is usually implemented by using phase modulation in Fourier domain. In this paper, we present a type of 4F system based on intensity modulation in Fourier domain as the measurement system for compressed sensing. The feasibility of this system is demonstrated. At the point of coherence, the two modulation methods are compared and superiority of intensity modulation in Fourier domain was verified. Simulations are presented and the conclusion we presented is validated. Finally, we analyze the results.

1. Introduction

Recently, compressed sensing (CS) theory was discussed widely [1–3]. Compared with the traditional sampling method, compressed sensing is based on the sparsity of signal. In the compressed sensing framework, we need to design a random matrix to measure signal and recover the signal by convex optimization. Actually, the measurement procedure can be understood as projecting the signal into a random vector of a measurement matrix. By using random measurement, CS can hugely reduce the sampling ratio which is lower than the Nyquist sampling ratio, as a result we save the time spent on the sampling and data size of the sampling value is smaller than before. Its appearance is very important to the signal process.

CS has been found to have wide usage in many applications, such as optical imaging [4,5], biomedical imaging [6], high-speed wireless communication [7], etc. In the imaging field, the single pixel camera based on compressed sensing has been presented by Davenport M.A. [8]. The advantage of single pixel camera is its suitability for optical systems. In the compressed imaging field, Rebecca M. Willett's group improved the image resolution by using compressed coded aperture [9]. Gonzalo R. Arceze presented the variable density compressed image sampling [10]. Justin Romberg proved that the measurement system consisted of the 4F system based on phase modulation in the Fourier domain can meet compressed sensing requirements [11].

Inspired by Justin Romberg, we present our purposed measurement system consisting of the 4F system based on intensity modulation in the Fourier domain and proves it to be fitted for compressed sensing. Another contribution of this paper is that compared to the measurement system using phase modulation in the Fourier domain [11], the superiority of our purposed system was verified at the point of coherence.

This work is organized as follows: Section 2 introduces the principle of compressed sensing. Section 3 contains the main theoretical work of this paper and we construct the 4F system based on intensity modulation in the Fourier domain as the measurement system. Section 4 is the numerical simulation, in which the feasibility of our imaging system is validated. Finally, Section 5 concludes this paper.

2. Principle of compressed sensing

The principle of compressed sensing can be expressed as follow. To recover the unknown signal $x \in R^n$, we need to measure it and the measurement results can be written as:

$$y_1 = \langle x, \phi_1 \rangle \cdots y_m = \langle x, \phi_m \rangle \quad (1)$$

ϕ_k is a measurement vector whose size is same as signal x. y_i is measurement value. We can use Φ as the measurement system, where the ϕ_k are stacked up as the rows in Φ. The measurement process can be expressed as:

$$y = \Phi x = \Phi \Psi \alpha \quad (x = \Psi \alpha). \quad (2)$$

In general, if one wants to recover signal x from y, we need $m \geq n$, but when signal x is sparse in an orthogonal basis Ψ (i.e. there are relatively few nonzero entries in α and the number of nonzero

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http://dx.doi.org/10.1016/j.ijleo.2014.01.123
0030-4026/

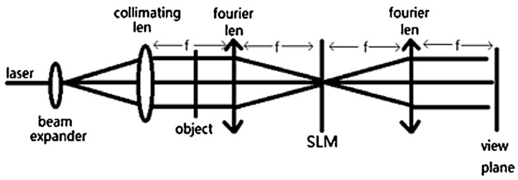

Fig. 1. Compressed sensing based on intensity modulation in 4f system.

entries is K, called K-sparse), then it is possible to reconstruct signal x with $m \ll n$.

For general images, there always is a space spanned by a proper orthogonal basis where the image signal is sparse. Aurelien Bourquard believes that the importance of Ψ is its mere existence [12]. Based on the sparse property, if matrix $\Theta = \Phi\Psi$ satisfies the RIP (restricted isometry property), the signal can be recovered accurately from the relatively fewer samples. The RIP is briefly reviewed later. Suppose $T \subset \{1, 2 \ldots N\}$ and $|T| \le K$, Φ_T which is indexed by T, is a subset of Φ. δ_k is the minimal constant which satisfies Eq. (4) and it is correlated to Φ_T and K. If there are Θ that meets Eq. (3), then we say that Θ satisfies RIP. From the point of RIP, the goal of the designing or optimizing measure system Θ is making δ_k as minimal as possible.

$$(1-\delta_k)||x||_2^2 \le ||\Theta_T x||_2^2 \le (1+\delta_k)||x||_2^2 \tag{3}$$

RIP is a sufficient but not necessary condition, and it is hard to determine whether or not the measurement system meets RIP by algorithm. In fact, to implement compressed sensing, one always designs the measurement system Θ which is incoherent or low coherent with Ψ. The coherence can be defined as follows. Suppose the measurement system is Θ, and the orthogonal basis is Ψ. The coherence μ can then be written as:

$$\mu = \max_{l,k} \left\langle \Theta_l, \Psi_k \right\rangle, \tag{4}$$

where Φ_l is the lth row in Φ and Ψ_k is the kth column in Ψ. In (4), μ satisfies $1 \le \mu \le \sqrt{N}$. Based on conditions before, recovering signal x is a convex optimization problem as shown in Eq. (5)

$$\min \left\| \Psi^T x \right\|_1 \quad s.t. \quad y = \Phi x. \tag{5}$$

3. System scheme

Romberg has proven that the 4F system using phase modulation in the Fourier domain can be incoherent with any orthogonal basis [11].

Based on Romberg's research, we present a type of 4F system based on intensity modulation in the Fourier domain as the measurement system. Section 3.1 introduces the composition of our imaging system. Sections 3.2 and 3.3 verify feasibility and superiority of our purposed measurement system respectively in theory.

3.1. Composition of imaging system

The basic composition of our purposed imaging system contains a measurement and sparse sampling system which are schematically presented in Fig. 1. The measurement system is composed of a pair of Fourier lenses and intensity SLM (spatial light modulator) and finishing modulation of input signal. The sparse sampling system randomly selects pixel locations and uses the pixel values of these positions as input signal for imaging system.

The process is shown as follows. At first, laser beam is expanded and collimated. A transmissive object is located in the way of beam. Intensity distribution after the object is regarded as the input signal for this imaging system. In the Fourier domain (i.e. the back focal plane of the first Fourier lens), the intensity SLM modulates the frequency of the input signal. On the imaging plane (i.e. the back focal plane of the second Fourier lens), the modulated image is sampled randomly. Finally, the original signal is reconstructed from the sampling result by using convex optimization.

The measurement system Φ in Fig. 1 is shown as follow:

$$\Phi = \sqrt{n} F^* P F, \tag{6}$$

where F is a two-dimensional discrete Fourier matrix which represents a two-dimensional discrete Fourier transformation (as shown in (7)), F^* is the conjugate matrix of F, and P (as shown in (8)) represents the intensity modulation in Fourier domain. The elements of P distribute uniformly and independently in the interval [0,1].

$$F(u, v) = \frac{1}{\sqrt{N}} e^{-i2\pi(u-1)(v-1)/N} \tag{7}$$

$$P = \begin{bmatrix} a_{11} & & & & \\ & \ddots & & & \\ & & a_{ii} & & \\ & & & \ddots & \\ & & & & a_{nn} \end{bmatrix} \quad a_{ii} \sim \text{uniform}[0, 1] \tag{8}$$

3.2. Incoherence analysis

Many works show that the key of implementing compressed sensing is designing a measurement system which is incoherent with any fixed orthogonal basis Ψ.

According to the definition of coherence in Eq. (4) and the conclusion in [11], when coherence satisfies Eq. (9), the measurement system is nearly incoherent with the orthogonal basis. As a result, we can reconstruct a signal at the smallest sampling ratio. With the coherence smaller than $2\sqrt{\log(4n^2/\delta)}$, the probability of $P(\max_{l,k} |\langle \Phi_l, \Psi_k \rangle| < 2\sqrt{\log(4n^2/\delta)})$ will exceed $1-\delta$ (as shown in (8)) and μ will approach 1 [11].

$$P(\max_{l,k} |\langle \Phi_l, \Psi_k \rangle| < 2\sqrt{\log(4n^2/\delta)}) > 1-\delta \tag{9}$$

Based on the analysis before, it is obvious that if the proposed measurement system can satisfy (8), then it will fit for compressed sensing.

According to the expression of matrix P in (7), the coherence can be written as:

$$\left\langle \Phi_l, \Psi_k \right\rangle = \sum_{j=1}^{n} e^{\frac{i2\pi(l-1)(j-1)}{n}} a_{jj} \hat{\Psi}_k(j) \tag{10}$$

In Eq. (9), $\widehat{\Psi_k}(j) = F\Psi_k$, $e^{(i2\pi(l-1)(j-1))/n} = i\theta$, Eq. (8) can be written as:

$$\left\langle \Phi_l, \Psi_k \right\rangle = \sum_{j=1}^{n} e^{i\theta} a_{jj} \hat{\Psi}_k(j) \tag{11}$$

In the complex plane, it is obvious that

$$|e^{i\theta}| = |\cos\theta + i\sin\theta| = 1 \tag{12}$$

$$\left|(\cos\theta + i\sin\theta) a_{jj} \hat{\Psi}_k(j)\right| \le \left|a_{jj}\right| \left|\hat{\Psi}_k(j)\right| \tag{13}$$

Because of $|a_{jj}| \in [0, 1]$, (13) can be written as:

$$\left|(\cos\theta + i\sin\theta) a_{jj} \hat{\Psi}_k(j)\right| \le \left|\hat{\Psi}_k(j)\right| \tag{14}$$

The complex Hoeffding inequality tells us that,

$$P\left(\left|\left\langle \Phi_l, \Psi_k \right\rangle\right| > \lambda\right) \le 4\exp\left\{-\frac{\lambda^2}{2\sum_{j=1}^{n} \left|\hat{\Psi}_k(j)\right|^2}\right\} \tag{15}$$