Optik 125 (2014) 3088–3092

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Focus evolution of linearly polarized Lorentz beam with sine-azimuthal variation wavefront induced by one on-axis optical vortex

Xiumin Gao [a,*], Quanyong Shen [b], Yanlin Zhang [a], Qing Xin [a], Xinmiao Lu [a], Qingling Qu [a]

[a] *Electronics and Information College, Hangzhou Dianzi University, Hangzhou 310018, China*
[b] *Zhejiang Industry Polytechnic College, Shaoxing 312000, China*

A R T I C L E I N F O

Article history:
Received 22 June 2013
Accepted 15 December 2013

Keywords:
Lorentz beam
Focal pattern
Optical vortex

A B S T R A C T

Focus evolution of linearly polarized Lorentz beam with sine-azimuthal variation wavefront induced by one on-axis optical vortex was investigated theoretically in this article. Calculation results show that the focal pattern can be altered considerably by the charge number of on-axis optical vortex under condition of certain beam parameters and phase parameter that indicates the sine phase change frequency on increasing azimuthal angle. And the focal evolution principle differs remarkably for different beam parameters and the phase parameter. In focus evolution process, some novel focal patterns appear, including annular focal pattern, two-peak focal pattern, intensity lines, hexagon containing two peaks, swallowtail shape, multipede shape, and complex focal pattern. Introduction of optical vortex adds one controllable parameter to alter focal pattern, which may extend application of Lorentz beam in some focusing systems.

1. Introduction

Since Omar El Gawhary and Sergio Severini introduced Lorentz beams [1–3], much attention was paid to the description and propagation of Lorentz beams [4–11]. Based on the second-order moments, the beam propagation factors and the kurtosis parameters of a Lorentz beam were investigated by Zhou [4], which shows that the M^2 value is verified to be $\sqrt{2}$ and kurtosis parameter varied upon the propagation. An analytical expression for a Lorentz beam passing through a fractional Fourier transform system was derived based on the definition of convolution and the convolution theorem of the Fourier transform, and the properties of Lorentz beam in the fractional Fourier transform plane were illustrated numerically [5]. In addition, Zhou also investigated the generalized M^2 factor of truncated partially coherent Lorentz and Lorentz–Gauss beams [6]. By expanding a Lorentz distribution in terms of orthogonal Hermite–Gauss beams, Zhou and co-workers obtained an analytical formula for the average intensity distribution on the basis of the extended Huygens–Fresnel principle, and found that the intensity pattern evolves from a Lorentz-shaped spot into a Gaussian-shaped spot under isotropic influence of the turbulence [7]. Propagation properties of a Lorentz beam array were studied in detail, and effect of phase errors on the far-field intensity pattern is also given [8]. Yu and co-workers have extended the Lorentz beams to the nonparaxial regime [9]. The properties of Lorentz beams propagating in uniaxial crystals orthogonal to the optical axis are studied [10], which shows that the transverse components would finally convert into a specific four-petal profile with an axial shadow after propagating for sufficient distances, and the Lorentz beam parameters and the ratios of refractive indices have strong influences on the diffraction field components and on the effective beam sizes when propagating in uniaxial crystals. By expanding the hard-aperture function into a finite sum of complex Gaussian functions, Du et al. derived the approximate analytical formulae for Lorentz beams propagating through an apertured fractional Fourier transform optical system [11]. Propagation of a radial phased-locked Lorentz beam array in turbulent atmosphere was also studied [12]. Recently, focusing of linearly polarized Lorentz beam with sine-azimuthal variation wavefront have been investigated, which show that the focal pattern can be altered considerably by the beam parameters and the phase parameter, and some novel focus shape may appear, including multiple-peak focal pattern, wheel focal pattern [13].

On the other hand, researches on optical vortices have grown rapidly recently because optical vortices have some interesting properties and promising applications [14–19]. For instance, optical vortices can be used to construct highly versatile optical tweezers and arrays vortices can also assemble micro-particles into dynamically optical pumps [20]. Recently, optical vortex was used in terabit free-space data transmission [21,22], quantum correlations [23],

* Corresponding author. Tel.: +86 13456936705.
E-mail address: gxm@hdu.edu.cn (X. Gao).

0030-4026/$ – see front matter © 2014 Elsevier GmbH. All rights reserved.
http://dx.doi.org/10.1016/j.ijleo.2014.01.017

and quantum entanglement [24]. However, to best of our knowledge, there is no report on the focusing properties of Lorentz beams with optical vortex to data. In order to get insight into the focusing properties of Lorentz beams, and extend their application, we investigate the focusing evolution of linearly polarized Lorentz beam with sine-azimuthal variation wavefront induced by one on-axis optical vortex. In Section 2, the focusing principle of the kind of Lorentz beams is given. Results and discussions are shown in Section 3. And conclusions are summarized in Section 4.

2. Principle of focusing sine-azimuthal Lorentz beam with one on-axis optical vortex

In the focusing system we investigated in this article, the incident laser beam is chosen as Lorentz beam, and its amplitude distribution of electric field is written in the form [1–5]

$$E(x,y) = A \times \frac{1}{\omega_x \times \omega_y} \times \frac{1}{1+\left(x/\omega_x\right)^2} \times \frac{1}{1+\left(y/\omega_y\right)^2} \quad (1)$$

where A is a constant value, ω_x and ω_x are parameters related to the beam width, with A, ω_x and $\omega_y \in \Re$. This kind of electric field is product of the two functions of parameter ω_x and ω_y. By the similar derivation method shown in reference [13], Eq. (1) in cylindrical coordinate system (r_0,φ) in pupil position can be rewritten as,

$$E(r_0,\varphi) = A \times \frac{1}{\omega_x \times \omega_y} \times \frac{1}{1+\cos^2(\varphi) \times \left(r_0/\omega_x\right)^2} \times \frac{1}{1+\sin^2(\varphi) \times \left(r_0/\omega_y\right)^2} \quad (2)$$

And the r_0/ω_x can be written as $r_0/\omega_x = \sin\left(\theta\right)/(\mathrm{NA} \times w_x)$, where $w_x = \omega_x/r_p$ is a relative beam waist in x coordinate direction. r_p is the outer radius of optical aperture in focusing system, f is focal length of the focusing system. NA is numerical aperture of the focusing system, defined as the multiple of refractive index of surrounding medium and sine value of aperture angle, $\mathrm{NA} = n\sin(\alpha) = r_p/f$, where n is refractive index of surrounding medium, the system we investigated is in air, so $n \approx 1$, α is aperture angle. As similar procedure, $r_0/\omega_y = \sin\left(\theta\right)/\left(\mathrm{NA} \times w_y\right)$, where $w_y = \omega_y/r_p$. It should be noted that the wavefront is the function of azimuthal angle $\phi = -\pi\sin(n\varphi)$ and contains one on-axis optical vortex $\exp(im\varphi)$, then the Eq. (2) can be written as,

$$E\left(\theta,\varphi\right) = \frac{C}{w_x \times w_y} \times \frac{1}{1+\cos^2(\varphi) \times \sin^2\left(\theta\right)/\mathrm{NA}^2 \times w_x^2} \times \frac{1}{1+\sin^2(\varphi) \times \sin^2(\varphi)/\mathrm{NA}^2 \times w_y^2} \times \exp\left[-i\pi\sin(n\varphi)\right] \times \exp(im\varphi) \quad (3)$$

where $C = A \times r_p^2$ is chosen as constant due to constant r_p in our investigations, n is phase parameter that indicates the sine phase change frequency on increasing azimuthal angle, and m is charge number of the optical vortex. It is assumed that the incident Lorentz beam is linearly polarized along the x axis. According to vector diffraction theory, the electric field in focal region can be written in the form [25,26],

$$\vec{E}\left(\rho,\psi,z\right) = \frac{1}{\lambda}\iint\limits_{\Omega}\left\{\left[\cos\theta+\sin^2\varphi\left(1-\cos\theta\right)\right]x+\cos\varphi\sin\varphi\left(\cos\theta-1\right)y+\cos\varphi\sin\theta z\right\}\frac{C}{w_x \times w_y} \times \frac{1}{1+\cos^2(\varphi) \times \sin^2\left(\theta\right)/\mathrm{NA}^2 \times w_x^2} \times \frac{1}{1+\sin^2(\varphi) \times \sin^2\left(\theta\right)/\mathrm{NA}^2 \times w_y^2} \times \exp\left[-i\pi\sin(n\varphi)\right] \times \exp(im\varphi) \times \exp\left[-ik\rho\sin\theta\cos\left(\varphi-\psi\right)\right] \times \exp\left(-ikz\cos\theta\right)\sin\theta d\theta d\varphi \quad (4)$$

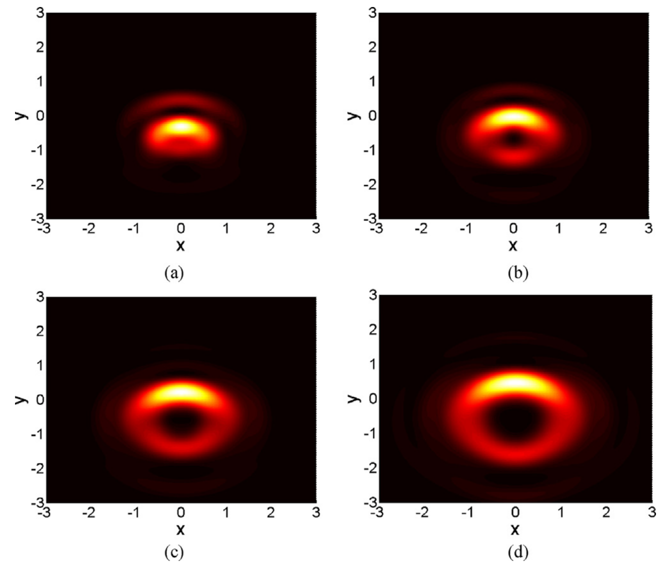

Fig. 1. Focal intensity distributions for NA = 0.95, n = 1, $w_x = w_y = 1$, and (a) m = 1, (b) m = 2 (c) m = 3, (d) m = 4, respectively.

where $\varphi \in [0,\ 2\pi)$, $\theta \in [0, \arcsin(NA)]$. Vectors x, y, and z are the unit vectors in the x, y, and z directions, respectively. It is clear that the incident beams is depolarized and has three components (E_i, E_j and E_k) in x, y, and z directions, respectively. The variables ρ, ψ, and z are the cylindrical coordinates of an observation point in focal region. The optical intensity in focal plane can be obtained by calculating the modulus square of Eq. (4).

3. Results and discussions

The intensity distributions are calculated and discussed here. It should be noted that the distance unit in all figures is wavelength of incident beam in vacuum. Firstly, the intensity distributions in focal region for NA = 0.95, n = 1, $w_x = w_y = 1$, and different m were calculated and illustrated in Fig. 1. It can be seen that the effect of optical vortex on focal pattern is very considerably, and on increasing charge number of the on-axis optical vortex, the whole focal pattern changes from one crescent shape to one annular shape, and the radius of the annular also increases at the same evolution. Although the total phase distribution of wavefront is rotational symmetric, the whole focal pattern is not rotational symmetric due to highly focusing of linearly polarized incident laser beam, under this condition, the vector characteristics should not be neglected. We can see that the focal pattern is also symmetry about y coordinate axis. In order to get more deeply insight into focusing properties, the intensity curves along y coordinate axis are shown in Fig. 2(a), from which it can be seen that the intensity maximum shift along y coordinate axis in positive direction on increasing charge number. The corresponding focal shift is also illustrated in Fig. 2(b), which draws the path of moving intensity maximum peak. Value of focal shift is wavelength of incident beam in vacuum space. It is clear that the dependence of focal shift on charge number is nearly liner with little curve.