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# Fourth order correction to a Gaussian beam focused with a parabolic index lens

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ABSTRACT

We formulate the fourth order correction to a paraxial Gaussian beam propagated along the axis of symmetry of a parabolic index lens. First we examine the evolution of a complex-source-point spherical wave (equivalent paraxially to a Gaussian beam) through the lens in a two-dimensional *xz* plane. Taking into account the terms of up to fourth order in aperture variables, we find a ray-optical solution to the exit beam that is represented in terms of aberration function. We also analyze the effect of the lens aberration exerted on the degradation in the quality of a Gaussian beam. The fourth order-corrected wave function derived here may be used to evaluate the quality of a Gaussian beam focused with a parabolic index lens. Further it may be applied to the case of an orthogonal system in which the index variations are different in the *xz* and *yz* planes.



## 1. Introduction

A laser beam is generally highly directional and does not have a spatially uniform intensity distribution. The laser beam of fundamental mode confined to a region near the optic axis is well-described as a Gaussian beam [1]. It is also known that the sum of all the higher order corrections to the paraxial Gaussian beam is equivalent to a spherical wave emanating from a complex source point [2,3]. Recently we have examined the evolution of a complex-source-point spherical wave (CSPSW) through a rotationally symmetric optical system which is composed of various refracting (or reflecting) surfaces such as spheres, aspherics, and zone plates [4–8]. As a result, fourth order corrections have been made in terms of Seidel-type aberration coefficients to the Gaussian beam passing through the optical system. We have also analyzed what aberrations of the system degrade the quality of the Gaussian beam. Today inhomogeneous or gradient index (GRIN) materials are finding a variety of applications as lenses in micro-optical systems and a substitute for aspherics in conventional imaging systems [9]. In particular, the parabolic index profile is worth considering in the context of GRIN lenses since it yields analytic results which facilitate the understanding of more general forms of profile gradient [10]. However, we have not discussed the quality of a Gaussian beam focused with a parabolic index lens.

In this article, we discuss the Seidel-type aberration of a parabolic index lens which degrades the quality of a Gaussian beam propagated along the axis of symmetry of the lens. First we examine the evolution of the equivalent CSPSW through the lens in a two-dimensional *xz* plane. Taking into account the terms of up to fourth order in aperture variables, we find a ray-optical solution to the exit beam that is represented in terms of aberration function. The real part of the aberration function has an analogy to the spherical wavefront aberration in ordinary ray optics [11], while its imaginary part gives the fourth order correction to the amplitude variation of the Gaussian beam. Using the derived formula, we also analyze the effect of the lens aberration exerted on the quality of a Gaussian beam. The fourth order-corrected wave function derived here may be used to evaluate the quality of a Gaussian beam focused with a parabolic index lens. Further it may be applied to the case of an orthogonal system in which the index variations are different in the *xz* and *yz* planes.

## 2. Evolution of a complex-source-point spherical wave through a parabolic index lens

Fig. 1 shows (a) a paraxial Gaussian beam of vacuum wavelength $\lambda$ which is focused with a parabolic index lens and (b) a complex-source-point spherical wave (CSPSW) which is equivalent paraxially to the Gaussian beam. $w_0$ (or $w'_0$) is the minimum spot size of the incident (or exit) Gaussian beam. The lens of thickness $\delta$ separates two media of refractive indices $n$ and $n'$. The coordinate system is referenced to the front surface of the lens which is

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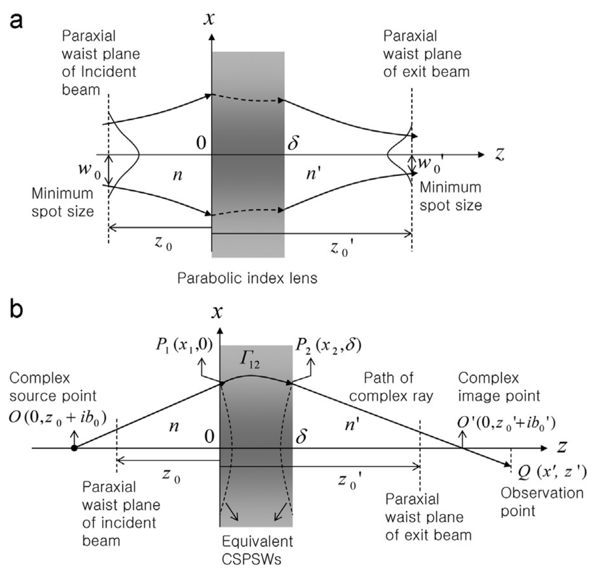


**Fig. 1.** (a) A paraxial Gaussian beam of vacuum wavelength $\lambda$ is propagated along the axis of symmetry of a parabolic index lens. The lens of thickness $\delta$ separates two media of refractive indices $n$ and $n'$. The coordinate system is referenced to the front surface of the lens. $w_0$ (or $w_0'$) is the minimum spot size of the incident (or exit) beam. (b) The Gaussian beam is equivalent paraxially to the spherical wave with a center at a complex location $(0, z_0 + ib_0)$ or $(0, z_0' + ib_0')$ in the $xz$ plane. The Rayleigh range of the incident (or exit) beam is given by $b_0 = n\pi w_0^2/\lambda$ (or $b_0' = n'\pi w_0'^2/\lambda$). The path of the complex ray is determined by applying Fermat's principle.

parallel to the $x$-and $y$-axes, and the axis of symmetry of the lens is taken as the $z$-axis. A ray of light starts from a source point $O$ on the $z$-axis, passes through the points $P_1$ and $P_2$ on the front and rear surfaces of the lens, and goes to an image point $O'$ on the $z$-axis. By rotational symmetry of the lens under consideration, the path of the ray from $O$ through $P_1$ and $P_2$ to $O'$ must always lie on one plane. Therefore, we may treat this problem in a two-dimensional space, chosen as the $xz$ plane. If the source point $O$ is denoted by coordinates $(0, z_0 + ib_0)$, the points $P_1$ and $P_2$ by coordinates $(x_1, 0)$ and $(x_2, \delta)$, and the image point $O'$ by coordinates $(0, z_0' + ib_0')$, the light disturbance arriving at $P_1$ from $O$ may be represented by

$$\psi(x_1, 0) = \frac{C}{r} \exp(iknr - i\omega t), \tag{1}$$

where $C$ is the normalization constant, $i(=\sqrt{-1})$ is the imaginary symbol, $k(=2\pi/\lambda)$ is the magnitude of the wave vector in vacuum, $\omega$ is the angular frequency of the light, and

$$r = \left[x_1^2 + (z_0 + ib_0)^2\right]^{1/2}. \tag{2}$$

In the above we choose the branch of $r$ such that its real part is equal to $-z_0$ when it is large. By so doing, the wave function in Eq. (1) is equivalent to the sum of all the higher order corrections to the paraxial Gaussian beam of Rayleigh range $b_0$, where $b_0 = n\pi w_0^2/\lambda$ [2,3].

If the medium of the lens has a dielectric function of parabolic type

$$\epsilon(x) = n_0^2(1 - Bx^2), \tag{3}$$

in the $xz$ plane [10], where $n_0$ is the index of refraction on the $z$-axis and $B(>0)$ is the parameter governing the index variation, the transmission coefficient for a ray of light impinging on $P_1$ may be expressed as follows:

$$\tau_{12} \simeq \tau_0 \exp(ik\Gamma_{12}) \tag{4}$$

where $\tau_0$ is the constant factor and $\Gamma_{12}$ is the optical path length of the ray from $P_1$ to $P_2$, evaluated as

$$\Gamma_{12} = \int_1^2 \sqrt{\epsilon(x)[(dx)^2 + (dz)^2]} = \frac{\delta}{2\beta_1}(n_0^2 + \beta_1^2) + \frac{2\epsilon(x_1) - n_0^2 - \beta_1^2}{4n_0\sqrt{B}} \sin\left(\frac{2n_0\delta\sqrt{B}}{\beta_1}\right) - \frac{nx_1^2}{r}\sin^2\left(\frac{n_0\delta\sqrt{B}}{\beta_1}\right), \tag{5}$$

in terms of the constant of motion in the $z$-direction

$$\beta_1 = \sqrt{\epsilon(x_1) - \left(\frac{nx_1}{r}\right)^2}. \tag{6}$$

On the one hand, in the paraxial regions such that $x_1^2 \ll |z_0 + ib_0|^2$ and $Bx_1^2 \ll 1$, the light disturbance transferred from the source point $O$ through the lens to an observation point $Q$ can be evaluated using the Fresnel–Kirchhoff diffraction integral [12]. If the terms of up to second order in the aperture variable $x_1$ are taken into account, the amplitude of the light at $Q$ of coordinates $(x', z')$ is given by

$$\psi'(x', z') \simeq C' \int_{\text{aperture}} dx_1 \exp[ik(nr + \Gamma_{12} + n'r')], \tag{7}$$

where $C'$ is the factor independent of $x_1$ and the path lengths are written as

$$r \simeq -(z_0 + ib_0) - \frac{x_1^2}{2(z_0 + ib_0)},$$

$$r' \simeq (z' - \delta) + \frac{(x' - x_2)^2}{2(z' - \delta)} \quad (|z' - \delta| \gg |x' - x_2|),$$

$$\Gamma_{12} \simeq n_0\delta + \left\{\frac{\sin(2\delta\sqrt{B})}{4n_0\sqrt{B}}\left[\frac{n^2}{(z_0 + ib_0)^2} - n_0^2 B\right] + \frac{n\sin^2(\delta\sqrt{B})}{z_0 + ib_0}\right\} x_1^2, \tag{8}$$

in a quadric approximation. It should be noted here that a time-harmonic factor $\exp(-i\omega t)$ has been dropped from Eq. (7) for simplicity. Assuming the size of aperture is large enough to accept the paraxial Gaussian beam, we analytically solve the diffraction integral (7) to get

$$\psi'(x', z') \simeq A \exp\left[in'k\frac{x'^2}{2(z' - z_0' - ib_0')}\right], \tag{9}$$

where $A$ is the factor independent of $x'$ and the paraxial beam parameters are defined by

$$\begin{pmatrix} \dfrac{n'x_2}{z_0' - \delta + ib_0'} \\ x_2 \end{pmatrix} = \begin{bmatrix} \cos\left(\delta\sqrt{B}\right) & n_0\sqrt{B}\sin\left(\delta\sqrt{B}\right) \\ -\dfrac{1}{n_0\sqrt{B}}\sin\left(\delta\sqrt{B}\right) & \cos\left(\delta\sqrt{B}\right) \end{bmatrix} \times \begin{pmatrix} \dfrac{nx_1}{z_0 + ib_0} \\ x_1 \end{pmatrix}, \tag{10}$$

with the equivalent power of the lens system

$$K = n_0\sqrt{B}\sin\left(\delta\sqrt{B}\right). \tag{11}$$

The wave function in Eq. (9) is equivalent paraxially to a spherical wave with a center at a complex location $(0, z_0' + ib_0')$. It also represents the exit Gaussian beam of Rayleigh range $b_0'$, where $b_0' = n'\pi w_0'^2/\lambda$.

On the other hand, if we regard the equivalent CSPSW in Eq. (1) as a bundle of rays originating from a complex source point $O$, the formula in Eq. (9) can be derived from Fermat's principle. The path