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Temporal diffraction characteristics of transmitted multilayer volume holographic grating illuminated by an ultrashort pulse

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ABSTRACT

Based on the coupled wave theory of Kogelnik and Fourier optics, the time-domain diffraction characteristics of tramsmitted multilayer volume holographic grating (MVHG) under an ultrashort pulse readout are investigated. It is shown that the temporal diffraction characteristics depend not only on the numbers of the grating layers, but also on the thicknesses of the grating layers and buffer layers, grating period and the refractive index modulation. Furthermore, using group velocity dispersion we give explanation on the time-delay of diffraction pulse with respect to the center of the readout pulse. Results of our discussion may find applications in optical communications, pulse shaping and processing.

1. Introduction

Due to high wavelength and angular sensitivity and high diffraction efficiency, volume holographic gratings (VHGs) have been widely used in data storage, optical communication and spectroscopy [1–3]. Lately, stratified holographic optical elements (SHOEs) [4] composed of a stack of thin or thick grating layers interleaved with optically homogeneous buffer layers were suggested. Thin SHOEs, working in Raman–Nath diffraction regime, have some similar properties to those of VHGs. Based on the thin SHOE, stratified waveguide grating filter [5], stratified waveguide grating coupler [6], multiple beam generator [7], holographic multiplexing and phase multiplexing [8] have been proposed.

Thick SHOEs or multilayer volume holographic gratings (MVHGs) consist of stacks of multiple volume gratings instead of thin gratings, which operate within the Bragg diffraction regime. Theory of MVHG is based on the coupled-wave theory of Kogelnik [9] and Matrix optics. Many scientists have studied on the theory of MVHG. Yakmovich [10] developed the multilayer coupled wave theory to study the multilayer systems. A closed-form expression was derived by Vre and Hesselink to describe the diffraction properties of layered transmission and reflection geometry of MVHG [11]. Zhang et al. [12] proposed a recursion formula for the reflectance of the stratified and phase-shifted volume index gratings and studied its applications on the group velocity controlling. With the development of the MVHG theory, applications such as wavelength division multiplexers and demultiplexers, dynamic multiple wavelength filter have been realized using the MVHG [13].

However, all above diffraction research are discussed when the MVHGs are illuminated by continuous wave. Ultrashort pulse can be thought of as a linear superposition of continuous waves. Diffraction of pulse from VHG is different much from that of continuous waves. Siiman et al. [14] reported on the diffraction characteristics of ultrashort laser pulse by volume Bragg gratings recorded in photo-thermo-refractive glass, and corrected the spatial-temporal distortions by using a volume grating pair. Yang et al. [15] studied on the instantaneous diffraction and transmission characteristics of photorefractive volume grating under ultrashort pulse readout. Wang et al. [16] studied the pulse shaping properties of VHGs recorded in the anisotropic media, then investigated the diffraction properties of the finite-sized anisotropic VHGs illuminated by ultrashort pulsed laser [17]. Till now, not much attention has been paid on the diffraction of the ultrashort laser pulse from MVHG. Yan et al. [18] have studied diffraction spectra of MVHG under ultrashort pulse readout in frequency domain. And here, we present an analysis on the time-domain diffraction characteristics of femtosecond pulse when it is diffracted by a MVHG.

2. The structure of MVHG

Fig. 1 shows the structure of MVHG, in which N thick volume grating layers are interleaved with $N-1$ optically homogeneous buffer layers. In general, all layers have different thickness, we represent T_i for VHG layers and d_i for buffer layers. For simplicity, the

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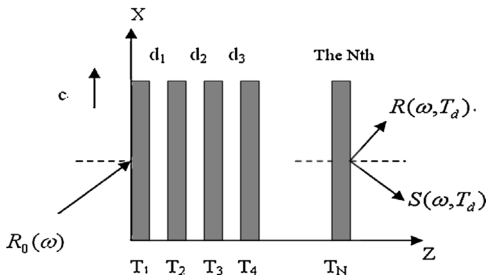

Fig. 1. Diffraction of multilayer volume gratings illuminated by an ultrashort pulse.

average refractive indices for both the grating layers and the buffer layers are set to be n_0 to neglect the multiple reflections at the different interfaces.

All VHGs are assumed to extent infinitely in the x-y plane and have the same grating material, grating orientation and grating period Λ. Parameters of the gratings are chosen to make the Q parameter, $Q = 2\pi T_i\lambda/(n_0\Lambda^2) \gg 1$ to guarantee that all gratings are thick, where λ is the readout wavelength.

Considering that all the VHGs are phase gratings, the corresponding refractive index for each grating layer is described by

$$n(\omega) = n_0(\omega) + n_1\cos(\vec{K}\cdot\vec{r}) \quad n_1 << n_0(\omega) \tag{1}$$

where n_1 is the amplitude of the modulated refractive index, $\omega = 2\pi c/\lambda$ is the angular frequency of the input pulse, c is the speed of light in free space, and $\boldsymbol{K}$ is the grating vector and the magnitude is $K = 2\pi/\Lambda$.

3. Theoretical model of MVHG

Supposing a single ultrashort Gaussian pulse $r_0(t) = \exp(-j\omega_0 t - t^2/\tau^2)$ is incident on the MVHG with angle θ at $z = 0$ to readout the recorded gratings, where ω_0 is the central angular frequency of the incident pulse. $\tau = \Delta\tau/2\sqrt{\ln 2}$, with $\Delta\tau$ the full width at half maximum (FWHM). In incident, the central frequency component of the readout pulse satisfies the Bragg condition of the volume gratings.

The Fourier transform of the incident pulse is

$$R_0(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} r_0(t)\exp(j\omega t)\mathrm{d}t = \frac{\tau}{2\sqrt{\pi}}\exp\left[-\frac{\tau^2(\omega-\omega_0)^2}{4}\right] \tag{2}$$

Due to Bragg diffraction, in the ith grating layer, there has two output beams, the transmission and the diffraction beams, the total field can be written as

$$E_i(z,\omega) = \hat{e}_r R_i(z,\omega)\exp(-j\vec{k}_r\cdot\vec{x}) + \hat{e}_s S_i(z,\omega)\exp(-j\vec{k}_s\cdot\vec{x}) \tag{3}$$

where $R_i(z,\omega)$, $S_i(z,\omega)$ and $\hat{e}_r$, $\hat{e}_s$ are the complex amplitudes and the unit polarization vectors of the transmission and diffraction beams in the ith grating layer. k_r, k_s are wave vectors of the transmission and diffraction beams. The phase-matching condition for Bragg diffraction is given by $k_s = k_r + K$.

Substituting Eqs. ((1),(3)) into the scalar wave equation $\nabla^2 E + k^2 E = 0$ $(k = 2\pi n/\lambda)$, using the slowly varying envelop approximation and ignoring the quadratic terms, we get the following coupled wave equations

$$\begin{aligned} R_i'(z,\omega) &= -j\nu S_i(z,\omega)\sqrt{\frac{C_S}{C_R}} \\ S_i'(z,\omega) + 2j\xi S_i(z,\omega) &= -j\nu R_i(z,\omega)\sqrt{\frac{C_R}{C_s}} \end{aligned} \tag{4}$$

where $C_R = \cos\theta$, $C_S = \cos\theta - K\cos\varphi/\beta$ are obliquity factors of the readout and diffraction beams, $\beta = 2\pi n_0/\lambda$ is the average propagation constant. φ is the slant angle of the grating fringes with respect to the normal of the grating material. $\nu = \pi n_1(\hat{e}_s\cdot\hat{e}_r)/[\lambda(C_R C_S)^{1/2}]$ is the coupling constant and Off-Bragg parameter is $\vartheta = K\cos(\varphi-\theta) - K^2c/2\omega n$, $\xi = \vartheta/2C_S$. When the phase matching condition $k_s = k_r + K$ is satisfied, we have $\xi = 0$.

By solving Eq. (4), we can get the following transfer matrix of VHG

$$\begin{bmatrix} R_{ir} \\ S_{ir} \end{bmatrix} = \begin{bmatrix} m_{i11} & m_{i12} \\ m_{i21} & m_{i22} \end{bmatrix} \times \begin{bmatrix} R_{il} \\ S_{il} \end{bmatrix} = [M_i]\begin{bmatrix} R_{il} \\ S_{il} \end{bmatrix} \tag{5}$$

where R_{ir}, S_{ir} and R_{il}, S_{il} are the amplitudes of the transmission and diffraction beams on the right and left hand boundaries of the ith grating layer, respectively. Where

$$\begin{aligned} m_{i11} &= \exp(-j\xi T_i)[\cos(\sqrt{\xi^2+\nu^2}T_i) + \frac{j\xi}{\sqrt{\xi^2+\nu^2}}\sin(\sqrt{\xi^2+\nu^2}T_i)], \\ m_{i12} &= -j\frac{\nu}{\sqrt{\xi^2+\nu^2}}\sqrt{\frac{C_S}{C_R}}\exp(-j\xi T_i)\sin(\sqrt{\xi^2+\nu^2}T_i), \\ m_{i21} &= -j\frac{\nu}{\sqrt{\xi^2+\nu^2}}\sqrt{\frac{C_R}{C_S}}\exp(-j\xi T_i)\sin(\sqrt{\xi^2+\nu^2}T_i), \\ m_{i22} &= \exp(-j\xi T_i)[\cos(\sqrt{\xi^2+\nu^2}T_i) - \frac{j\xi}{\sqrt{\xi^2+\nu^2}}\sin(\sqrt{\xi^2+\nu^2}T_i)]. \end{aligned} \tag{6}$$

The propagation of the transmission and diffraction beams along the ith buffer layer situated between two successive grating layers can be represented by the introduction of a buffer transfer matrix

$$\begin{bmatrix} R_{(i+1)l} \\ S_{(i+1)l} \end{bmatrix} = [D_i] \times \begin{bmatrix} R_{ir} \\ S_{ir} \end{bmatrix} \tag{7}$$

where

$$\begin{aligned} [D_i] &= \begin{bmatrix} \exp(-jk_{rz}^{bu}d_i) & 0 \\ 0 & \exp(-jk_{sz}^{bu}d_i) \end{bmatrix} \\ &= \exp(-jk_{rz}^{bu}d_i)\begin{bmatrix} 1 & 0 \\ 0 & \exp(-2j\zeta d_i) \end{bmatrix} \end{aligned} \tag{8}$$

In the buffer layer, we define a detuning parameter ζ

$$\zeta = \frac{k_{sz}^{bu} - k_{rz}^{bu}}{2} \tag{9}$$

The complete MVHG consists of a succession of N grating layers and $N-1$ intermediate buffer layers. On the input surface of $z = 0$, the boundary conditions are $S(0,\omega) = 0$ and $R(0,\omega) = R_0(\omega)$, the matrix equation for the whole MVHG is

$$\begin{bmatrix} R(T_d,\omega) \\ S(T_d,\omega) \end{bmatrix} = [M_c] \times \begin{bmatrix} R(0,\omega) \\ S(0,\omega) \end{bmatrix} \tag{10}$$

where $R(T_d,\omega)$ and $S(T_d,\omega)$ are transmission and diffraction field amplitudes at the output surface of the MVHG, with T_d the total thickness of the MVHG

$$T_d = \sum_{i=1}^{N} T_i + \sum_{i=1}^{N-1} d_i \tag{11}$$

And the transfer matrix is

$$[M_c] = M_N D_{N-1} M_{N-1} \ldots D_i M_i \ldots D_1 M_1 \tag{12}$$

From Eq. (10), the spectrum distribution $S(T_d,\omega)$ of the diffraction beam can be acquired. Inverse Fourier transform of $S(T_d,\omega)$,