Optik 125 (2014) 3299–3302

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Bright and dark solitons in optical metamaterials

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ARTICLE INFO

Article history:
Received 4 July 2013
Accepted 18 December 2013

OCIS:
060.2310
060.4510
060.5530
190.3270
190.4370

Keywords:
Kerr law
Solitons
Integrability
Metamaterials

ABSTRACT

This paper addresses the dynamics of solitons in optical metamaterials. Both bright and dark soliton solutions are obtained. The ansatz method of integration is employed to extract the 1-soliton solutions to the governing equations. A couple of constraint relations are obtained in order for these solitons to exist. A few numerical simulations are also given to expose the dissipative effects.

1. Introduction

Electromagnetic properties of complex materials with simultaneous negative and real dielectric permittivity (ϵ) and magnetic permeability (μ) have attracted a lot of attention in research [1–15]. Russian physicist Vaselago predicted that electromagnetic wave propagation in these media should give rise to several peculiar characteristics [14]. These media typically referred to as left-handed (L-H) media possess interesting features that may lead to unconventional phenomena in guidance, radiation and scattering of electromagnetic waves. Even though not found in nature, the novel and interesting features of these engineered materials and their possible applications to support short duration soliton and non-soliton pulses are the primary motivation of this research work. A clear distinction is present in terms of single negative such as negative refraction and with double negative (DNG) material [4,11,13,14]. Regular photonic crystal often time shows the negative refraction for the optical wave. For both cases, the optical wave encounter higher degree of losses. Even the soliton pulses which are evolved due to delicate balance between dispersion and nonlinearity will be dissipative in nature. Loss compensation is a challenge to engineer these types of materials.

Recently reported DNG materials in visible infrared region by Shalaev and others have shown promise to make optical waveguide with these materials. We use the dispersion profile of the reported metamaterial to determine the nature of the soliton pulse. The effect of loss has also been considered. Comprehensive analytical and numerical studies using split step Fourier method (SSFM) have been conducted to treat soliton wave propagation in regular positive index materials. Negative index materials attract interest in the nonlinear domain as it enhances the nonlinearity due the confinement of electric field in a small region and other multiple frequencies can be generated due to efficient phase matching process. We, then, extend the study for negative-indexed materials. Fig. 1 shows dissipative soliton wave propagating through the DNG material. High losses in DNG material cause the dissipation of the soliton.

Fig. 2 shows the 3D view of bright and dark soliton pulses propagating through 0.125 m optical waveguide with DNG metamaterial. The pulse was launched with 1.55 μm telecommunications wavelength. Because of high loss in the DNG material, the soliton pulse

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http://dx.doi.org/10.1016/j.ijleo.2013.12.061

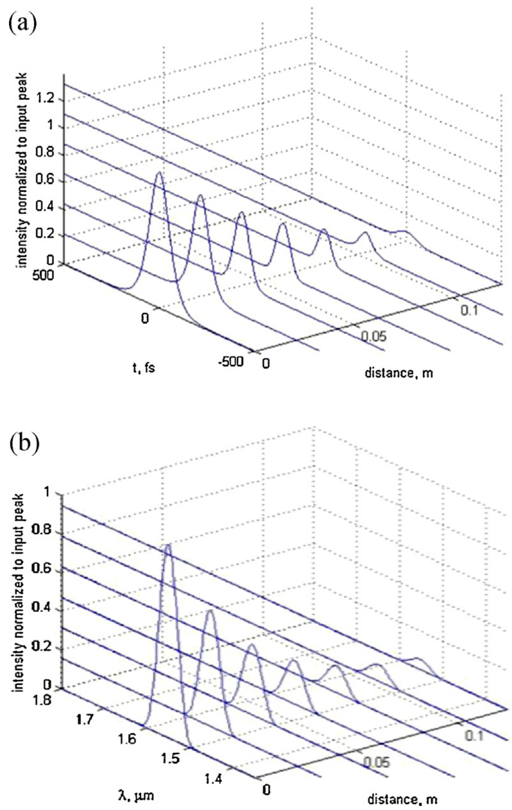

Fig. 1. Dissipative femto-second soliton pulse propagated through bulk DNG material (a) temporal view and (b) spectral view.

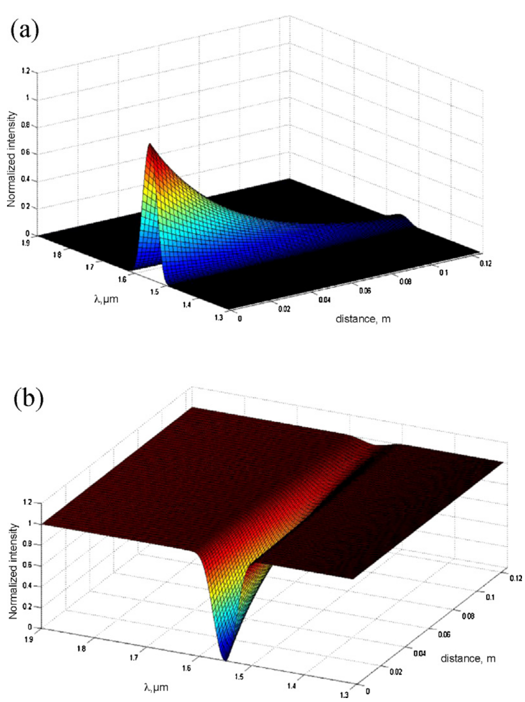

Fig. 2. 3D view of guided optical soliton pulses after propagating through a 0.125 m optical waveguide with meta materials: (a) bright soliton and (b) dark soliton.

gets attenuated over the distance. The shape of the soliton is conserved as long as the balance between power and dispersion is maintained.

2. Governing equation and soliton solutions

The dynamics of solitons in optical metamaterials is governed by the nonlinear Schrödinger's equation (NLSE) which in the dimensionless form is given by [15]

$$iq_t + aq_{xx} + b|q|^2 q = i\alpha q_x + i\lambda(|q|^2 q)_x + i\nu(|q|^2)_x q + \theta_1(|q|^2 q)_{xx} + \theta_2 |q|^2 q_{xx} + \theta_3 q^2 q^*_{xx} \quad (1)$$

Eq. (1) is the NLSE that is studied in the context of metamaterials. Here in (1), a and b are the group velocity dispersion and the self-phase modulation terms respectively. This pair produces the delicate balance between dispersion and nonlinearity that accounts for the formation of the stable solitons. On the right hand side λ represents the self-steepening term in order to avoid the formation of shocks and ν is the nonlinear dispersion, while α represents the intermodal dispersion. Then finally, θ_j for $j = 1, 2, 3$ are the perturbation terms that appears in the context of metamaterials [1].

This governing NLSE given by (1) will be solved by the aid of ansatz method. The bright and dark soliton solutions will be derived and discussed in the following two subsections. In order to proceed with the soliton solution, the following ansatz is adopted [1]

$$q(x, t) = P(x, t)e^{i\phi}, \quad (2)$$

In (2), $P(x, t)$ represents the bright or dark solitary wave profile and $\phi(x, t)$ is the phase component of the soliton that is defined as

$$\phi = -\kappa x + \omega t + \theta \quad (3)$$

where κ gives the soliton frequency and ω being the soliton wave number while θ represent the phase constant. Substituting (2) into (1) and then decomposing into real and imaginary parts leads to

$$(\omega + \alpha\kappa + a\kappa^2) + \{\kappa(\lambda - \theta_1\kappa^2 - \theta_2\kappa^2 - \theta_3\kappa) - b\}P^3 - a\frac{\partial^2 P}{\partial x^2} + 6\theta_1\left(\frac{\partial P}{\partial x}\right)^2 + (3\theta_1 + \theta_2 + \theta_3)P^2\frac{\partial^2 P}{\partial x^2} = 0 \quad (4)$$

and

$$\frac{\partial P}{\partial t} - (\alpha + 2a\kappa)\frac{\partial P}{\partial x} = (3\lambda + 2\nu - 6\theta_1\kappa - 2\theta_2\kappa + 2\theta_3\kappa)P^2\frac{\partial P}{\partial x} \quad (5)$$

The imaginary part leads to the relations

$$\nu = -\alpha - 2a\kappa \quad (6)$$

and

$$3\lambda + 2\nu - 2\kappa(3\theta_1 + \theta_2 - \theta_3) = 0 \quad (7)$$

Eq. (6) is the velocity of the soliton while relation (7) is the constraint condition that must be valid in order for the solitons to exist. These relations (6) and (7) remain valid for both bright and dark solitons. The real part equation, given by (4) will now be analyzed individually for bright and dark solitons in the following two subsections

2.1. Bright solitons

For bright solitons, the choice for the wave profile given by [1]

$$P(x, t) = A\,\mathrm{sech}^p\,\tau \quad (8)$$