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Shell histogram equalization of color images

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ABSTRACT

Histogram equalization (HE) is an effective technique for image enhancement. In this study, we devised a new technique called shell histogram equalization for color images. The technique is a dimensionality reduction method, which transforms 3-D space enhancement to 1-D shell enhancement. First, the 3-D RGB color space is decomposed into L ($L = 256$) RGB shells, which are similar to a quarter sphere shells or a quarter onion squamae. Then, HE is implemented on shells, and makes the shells coincide with the distribution of the iso-luminance-planes in the RGB cube. After analyzing the computational complexity of the proposed method, comparison experiments are carried out and validated by subjective and objective assessments. The experimental results show that the method provides better enhancement for underexposed and high dynamic range images, and the computational time of the method is much lower.

1. Introduction

The usage of digital images has rapidly increased with growing public consumption of entertainment and communication appliances. The expectation of a higher image quality prompts researchers to develop techniques for image enhancement. Histogram equalization (HE) has been one of the most widely used techniques due to its efficiency and simplicity in contrast enhancement.

Many methods have been developed for gray-level image improvement [1–3]. However, HE becomes a difficult task when dealing with color images owing to their multi-dimensional nature. Historically, the first extension of HE to color images was the application of HE to the red (R), green (G) and blue (B) components separately to achieve three transformed sub-images. The three gray sub-images were then composed into a color image. However, this method suffers from the presence of "blue" deflection [4–6]. Trahanias and Venetsanopoulos [5] proposed a "3-D histogram equalization" method, which will be discussed in Section 2. This method applies color modification along the $(1, 1, 1)^T$ orientation [5,6]. However, for a 3-D equalized result, the images usually become bright and tend to be faint which has been analyzed in [6].

Some new approaches have been proposed. Ref. [6] proposed a novel 3-D color HE method, which was implemented by defining an iso-luminance-plane cumulative distribution function (CDF) in the RGB color space. Kim and Yang interpolated the discrete probability density function (PDF) with Gaussian functions and applied nonlinear optimization [7]. Menotti et al. [8] partially overcame the over-enhancement of the 3-D HE by defining a new CDF that was a product of the marginal PDFs of each color channel.

There are many other color HE methods that are not directly related to the 3-D histogram. Ref. [9] proposed a 3-D approach called "histogram explosion" to counteract "white" faint in [5]. For each point in the RGB space, a ray that starts from the central point (which is usually chosen as the average color of the image) is defined to pass through that point, and all points within a threshold distance of the ray are projected on to the ray. Then, a 1-D histogram along the ray is implemented. Refs. [10,11] proposed a histogram decimation technique. The technique is outlined as follows. First, find the centroid of the image pixels. Second, shift the color values so that the centroid moves to the geometric center of the color space. Next, divide the current color space into eight equal-sized subspaces and partition the image color vectors among these subspaces. Any color vector that was shifted outside the current subspace will be assigned to the nearest new subspace. For each subspace, proceed recursively from the beginning. The recursion proceeds until the subspace decreases in size to a single color value. At last, all the color pixels in that subspace are shifted to that single color value.

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Traditional HE algorithms always incur "spikes" or "gaps". Histogram smoothing techniques that redistributed spikes and filled gaps to achieve a uniform PDF were implemented in [12–14], where Refs. [12,13] were implemented for gray images, Ref. [14] is devised for color images.

The aforementioned global methods always suffer from abrupt intensity changes over homogeneous regions. Ref. [4] proposed an adaptive or local HE method, which HE method was implemented at local image regions. Ref. [15] developed continuous distribution method to resolve the discontinuities in the intensity distribution.

This study focuses on a global 3-D histogram method. A descending dimension technique that transforms space enhancement to shell enhancement is implemented. HE is implemented on shells, which transforms a 3-D HE to a 1-D HE. The proposed shell histogram equalization (SHE) method is compared with conventional 3-D HE methods in human vision and objective assessment. We also calculate the processing time to evaluate the efficiency of the proposed method.

The rest of the paper is structured as follows. In Section 2, works related to 3-D color histogram, including methods proposed in [5,6,11], are outlined. In Section 3, the low-complex SHE method is presented. Experimental results are demonstrated in Section 4. Section 5 presents our discussions and conclusions.

2. Related works

The main idea of HE methods is to map the original image to the transformed image as $v = t(u)$. The main task of the map is to reassign the variable values of pixels to obtain an ideal enhancement without modifying its geometrical content. For n-dimensional HE, let $u = I(x, y)$, $v = I_o(x, y)$ and $u, v \in R^n$, $n = 1, 2, 3$, where x, y are indexes that run over the image, $I(x, y)$ is the pixel value at (x, y) of the original image, $I_o(x, y)$ is the output pixel value after enhancement. Then, a usual integral transformation $t : u \to v$ is constructed as follows:

$$\int_{\Omega_{out}(v)} f(x)dx = \int_{\Omega_{in}(u)} p(x)dx, \tag{1}$$

where $\Omega_{out}(v)$ and $\Omega_{in}(u) \subset R^n$ are the integral regions corresponding to v and u, respectively. Here, $p(x)$ is the PDF of the original image, $f(x)$ is the ideal probability density function (IPDF), i.e., the ideal PDF.

2.1. 3-D HE

From Eq. (1), the 3-D HE [5] formula can be written as

$$\int_0^{v_1} dx_1 \int_0^{v_2} dx_2 \int_0^{v_3} f(x_1, x_2, x_3)dx_3 = \int_0^{u_1} dx_1 \int_0^{u_2} dx_2 \int_0^{u_3} p(x_1, x_2, x_3)dx_3. \tag{2}$$

For any pixel value $u = (u_1, u_2, u_3)^T$ in the original image, the aforementioned integral equation maps output pixel values $v = (v_1, v_2, v_3)^T$. Obviously, the mapping maybe be non-injective, i.e., a single u maps out many v. Considering the uniqueness, v is usually searched along the main diagonal of the color space.

For the discrete case, when a uniform 3-D PDF $f(X) = 1/L^3$ ($L = 256$) is used as the IPDF, Eq. (2) is transformed as follows:

$$\frac{(k'+1)(s'+1)(t'+1)}{L^3} \geq \sum_{i=0}^{k}\sum_{j=0}^{s}\sum_{m=0}^{t} p(x_{R_i}, x_{G_j}, x_{B_m}), \tag{3}$$

where $k, s, t = 0, 1, \ldots, L-1$ are the inputs; $k', s', t' = 0, 1, \ldots, L-1$ are the outputs; $p(x_{R_i}, x_{G_j}, x_{B_m}) = N(x_{R_i}, x_{G_j}, x_{B_m})/N$ is the normalized original histogram, i.e., the probability mass function; N is the total pixel number in the image; $N(x_{R_i}, x_{G_j}, x_{B_m})$ is the number of pixels such that $I(x, y) = (x_{R_i}, x_{G_j}, x_{B_m})^T$; and $(x_{R_i}, x_{G_j}, x_{B_m})^T$ is the observation of the discrete random vector $X = (X_R, X_G, X_B)^T$, which has ith, jth and mth gray level in the RGB space, respectively. X_R, X_G, X_B models the R, G and B components of the original image in the RGB space.

Eq. (3) represents the 3-D HE method proposed by Trahanias and Venetsanopoulos [5], where the smallest triplet $(k', s', t')^T$ is chosen. They are repeatedly incremented (or decremented), one at a time, until Eq. (3) is satisfied.

2.2. RGB component margin method

Menotti et al. introduced a hue-preserving HE method based on the RGB space [11]. The 3-D HE method [5] has the disadvantage of tri-cycles, which is used to sum up the CDF. The method proposed in [8] evaluates CDF directly as follows:

$$C_{in}(x_{R_k}, x_{G_s}, x_{B_t}) = C_{x_{R_k}} \times C_{x_{G_s}} \times C_{x_{B_t}}, \tag{4}$$

where $C_{x_{R_k}} = \sum_{X_R \leq k} p(X)$, which denotes the margin cumulative distribution of the R component, and $C_{x_{G_s}}, C_{x_{B_t}}$ are illuminated similarly. For computational reduction, $C_{x_R} = (C_{x_{R_0}}, C_{x_{R_1}}, \ldots, C_{x_{R_{L-1}}})$, $C_{x_G} = (C_{x_{G_0}}, C_{x_{G_1}}, \ldots, C_{x_{G_{L-1}}})$ and $C_{x_B} = (C_{x_{B_0}}, C_{x_{B_1}}, \ldots, C_{x_{B_{L-1}}})$ are computed before the tri-cycle. For $k, s, t \in \{0, 1, \ldots, L-1\}$, the 3-D HE technique is implemented as follows:

$$\frac{(k'+1)(s'+1)(t'+1)}{L^3} \geq C_{in}(x_{R_k}, x_{G_s}, x_{B_t}), \tag{5}$$

where the smallest triplet $(k', s', t')^T$ is chosen similar to [5].

2.3. Iso-luminance-plane method

The iso-luminance-plane method in which the 1-D HE technique was implemented on luminance planes was proposed in [6]. These luminance planes are defined as $(X_R + X_G + X_B)/3 = n_k$, $n_k = 0, 1, \ldots, L-1$. Then, the 3-D CDF of the original image is defined as $C_{in}(n_k) = \sum_{X_R+X_G+X_B \leq n_k} p(X)$. The IPDF is uniform over all values of n, and the output CDF is given by $C_{out}(n_o) = n_o/L$, where n_o is the output luminance corresponding to n_k. From the 1-D HE principle, it can be inferred that $n_o = LC_{in}(n_k)$. Then a hue preserved technique based on luminance n_o, borrowed from [16], is implemented to complete pixel mapping.

3. Shell histogram equalization (SHE)

3.1. Mathematical description

From surface integral theory of the scalar field, surfaces can be integrated into a volume. Considering the color space topological configuration, the 3-D volume of $[0, L-1] \times [0, L-1] \times [0, L-1]$ can be integrated as $V = \int_0^{L-1} s_{\Sigma(t)} dt = \int_0^{L-1} 3t^2 dt = (L-1)^3$, where s is the area of $\Sigma(t)$, $\Sigma = \Sigma_1 \cup \Sigma_2 \cup \Sigma_3$, $\Sigma_i(t) = \{(x_1, x_2, x_3) | x_i = t; 0 \leq x_j \leq t; i, j = 1, 2, 3; j \neq i\}$. In the discrete RGB space, $\Sigma(t)$, $t = 1, 2, \ldots, L-1$ are called as *RGB shells* (Fig. 1(a)).

From Eqs. (1) and (2), the multi-dimensional HE method is multi-integral in nature. Multi-integral can be represented in different forms by integral region transformation. The SHE method, which transforms the 3-D HE to 1-D HE via integral region transformation, can be performed by Eq. (6) as follows:

$$\int_0^{v} f(t)dt = \int_0^{u} p_{\Sigma}(t)dt, \tag{6}$$