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Theoretical expressions of temporal power spectra of irradiance fluctuations for optical waves propagating through weak non-Kolmogorov turbulence

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ABSTRACT

The temporal power spectra of irradiance fluctuations reflect the frequency distribution of temporal statistical property of irradiance fluctuation. In this paper, new analytical expressions of the temporal power spectral models of irradiance fluctuations are developed for optical waves propagating through weak non-Kolmogorov turbulence with horizontal path. They are derived with the general modified atmospheric spectral model, and they consider the finite turbulence inner and outer scales, and have a general spectral power law value in the range of 3 to 4 instead of the standard power law value of 11/3. Numerical calculations are conducted to analyze the influence of non-Kolmogorov weak turbulence on the temporal power spectra of irradiance fluctuations.

1. Introduction

The irradiance fluctuations of an optical wave, as one of the atmospheric turbulence effects when optical wave propagating through the turbulence medium, degrade the performances of optical communication and imaging systems. It is hard to conduct experimental measurements of the spatial distribution property of turbulence effects because of the characteristics of time-varying of atmospheric turbulence. Therefore, in most applications, the temporal statistics of turbulence effects are actually measured. And then the spatial statistics of turbulence effects can be obtained by the relation between temporal and spatial statistics of turbulence effects with the Taylor's frozen hypothesis [1]. It permits converting spatial statistics into temporal statistics by the knowledge of the average wind speed transverse to the direction of observation.

However, the models derived for Kolmgorov turbulence cannot be applied directly in the non-Kolmgorov turbulence case. Compared with Kolmogorov turbulence, non-Kolmogorov turbulence covers a more wide range of atmospheric layers and this has been demonstrated by experimental results [2–5] and theoretical investigations [6,7]. Non-Kolmogorov spectral model has been used to investigate the temporal power spectra of irradiance fluctuations for plane and spherical waves propagating through weak non-Kolmogorov turbulence [8]. However, they have not considered the influences of turbulence inner and outer scales. As the irradiance fluctuations are mainly caused by the small scale turbulence cells' diffractive effects, the turbulence inner scale plays very important role in the analysis of irradiance fluctuations [9–12]. Compared with the generalized Von Karman [13] and the generalized exponential [14] spectral models, the generalized modified atmospheric spectral model [15] can feature the high frequency enhancement property (also called bump property, which is caused by the turbulence inner scale) in the irradiance fluctuations. This spectral model has been used to analyze the irradiance scintillation index (spatial property of irradiance fluctuations) for optical wave propagating through weak non-Kolmogorov turbulence [16,17].

In this study, the generalized modified atmospheric spectrum is adopted to investigate the temporal power spectra of irradiance fluctuations for optical waves propagating through weak non-Kolmogorov atmospheric turbulence. And then, the impacts of the turbulence inner scale values and the spectral power law values on the temporal power spectra of irradiance fluctuations are analyzed.

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2. Generalized modified atmospheric spectral model for non-Kolmogorov turbulence

The generalized modified atmospheric spectrum [15] is a non-Kolmogorov turbulence spectral model which considers the influence of finite inner and outer scales and has a general spectral power law value in the range of 3 to 5 instead of standard power law value 11/3 for Kolmogorov turbulence. Specifically, it takes the form as [15]

$$\Phi_n(\kappa,\alpha,l_0,L_0)=\hat{A}(\alpha)\times\hat{C}_n^2\times\kappa^{-\alpha}\times f(k,l_0,L_0,\alpha)\quad(0\le\kappa<\infty,\quad 3<\alpha<5),\tag{1}$$

$$f(\kappa,l_0,L_0,\alpha)=\left[1-\exp\left(-\frac{\kappa^2}{\kappa_0^2}\right)\right]\left[1+a_1\times\left(\frac{\kappa}{\kappa_l}\right)-b_1\times\left(\frac{\kappa}{\kappa_l}\right)^{\beta}\right]\exp\left(-\frac{\kappa^2}{\kappa_l^2}\right).\tag{2}$$

where α is the general spectral power law, $\hat{A}(\alpha)$ is a constant which maintains consistency between the refractive index structure function and its power spectrum, κ is the angular spatial frequency with units of rad/m, $\kappa_l=c(\alpha)/l_0$, $\kappa_0=C_0/L_0$. l_0 and L_0 are turbulence inner and outer scales, respectively. The choice of C_0 depends on the specific application, and it is set to 4π in this study just as [12]. $\hat{C}_n^2$ is the generalized refractive-index structure parameter with unit $m^{3-\alpha}$. The coefficients of a_1, b_1 and β in the generalized modified atmospheric spectrum depend on experimental results. $\hat{A}(\alpha)$ and $c(\alpha)$ have the form as [15]

$$\hat{A}(\alpha)=\frac{\Gamma(\alpha-1)}{4\pi^2}\sin\left[(\alpha-3)\frac{\pi}{2}\right],\tag{3}$$

$$c(\alpha)=\left\{\pi\hat{A}(\alpha)\left[\Gamma\left(-\frac{\alpha}{2}+\frac{3}{2}\right)\left(\frac{3-\alpha}{3}\right)+a_1\times\Gamma\left(-\frac{\alpha}{2}+2\right)\left(\frac{4-\alpha}{3}\right)-b_1\times\Gamma\left(\frac{-\alpha+3+\beta}{2}\right)\left(\frac{3+\beta-\alpha}{3}\right)\right]\right\}^{1/\alpha-5}.\tag{4}$$

when $\alpha=11/3$, the generalized modified atmospheric spectral model is reduced to the Kolmogorov modified spectral model, and when $l_0\to 0$, $L_0\to\infty$, it becomes the general non-Kolmogorov spectral model

$$\Phi_n=\hat{A}(\alpha)\times\hat{C}_n^2\times\kappa^{-\alpha}\quad(0\le\kappa<\infty,\quad 3<\alpha<5)\tag{5}$$

3. Temporal power spectra of irradiance fluctuations for non-Kolmogorov turbulence

Following Tatarski [1], the temporal power spectra of irradiance fluctuations, or power spectral density (PSD) $W_I(\omega)$, is defined by the Fourier transform of the temporal covariance function of irradiance fluctuations $C_I(t)$ according to [1]

$$W_I(\omega)=4\int_0^{\infty}C_I(t)\cos(\omega t)\,dt.\tag{6}$$

With the Taylor frozen turbulence hypothesis, $C_I(t)$ can be determined from the spatial covariance function of irradiance fluctuations $C_I(\rho)$. And $C_I(\rho)$ is given as [1]

$$C_{I(pl)}(\rho)=8\pi^2k^2\int_0^L\int_0^{\infty}\kappa\Phi_n(\kappa)J_0(\rho\kappa)\left[1-\cos\left(\frac{\kappa^2(L-z)}{k}\right)\right]d\kappa dz,\tag{7}$$

$$C_{I(sp)}(\rho)=8\pi^2k^2\int_0^L\int_0^{\infty}\kappa\Phi_n(\kappa)J_0(\rho\kappa)\left[1-\cos\left(\frac{\kappa^2z(L-z)}{Lk}\right)\right]d\kappa dz.\tag{8}$$

where ρ represents the geometrical separation between points in the plane transverse to the direction of propagation, $k=2\pi/\lambda$ and λ denotes the optical wavelength. $J_0(\rho\kappa)$ denotes the zero and second order Bessel functions, respectively. The Taylor frozen turbulence hypothesis make the association of $\rho=\nu_\perp t$ satisfied, where $\nu_\perp$ denotes the wind velocity perpendicular to the optical wave propagation path. In this case, $C_I(t)$ can be written as

$$C_{I(pl)}(t)=8\pi^2k^2\int_0^L\int_0^{\infty}\kappa\Phi_n(\kappa)J_0(\nu_\perp t\kappa)\left[1-\cos\left(\frac{\kappa^2(L-z)}{k}\right)\right]d\kappa dz,\tag{9}$$

$$C_{I(sp)}(t)=8\pi^2k^2\int_0^L\int_0^{\infty}\kappa\Phi_n(\kappa)J_0(\nu_\perp t\kappa)\left[1-\cos\left(\frac{\kappa^2z(L-z)}{Lk}\right)\right]d\kappa dz.\tag{10}$$

To consider the influences of finite turbulence inner and outer scales and general spectral power law values, in the next section, $W_I(\omega)$ will be replaced by $W_I(\omega,\alpha,l_0,L_0)$ and analytical expressions of temporal power spectra of irradiance fluctuations will be derived for plane and spherical waves propagating through weak non-Kolmogorov turbulence with horizontal path.