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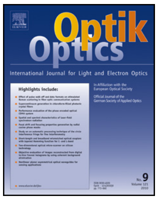

Relative phase based manipulation of quantum entanglement and measurement of supercurrent

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ABSTRACT

We investigate the quantum entanglement and supercurrent of coupling superconducting qubits in circuit QED system. We compare the effect of the relative phase of the coupling qubits on the concurrence and supercurrent when the microwave field is initially in coherent state, even coherent state and odd coherent state. The results show that entanglement death can be avoided via manipulating the relative phase only in the coherent state since the improvement for entanglement death is unsatisfactory in the even coherent state and odd coherent state.

1. Introduction

Circuit-QED system is the realization of cavity QED principle in solid field. In circuit-QED, superconducting qubits act as the artificial atoms while the dimensional superconducting transmission linear resonator acts as the microwave cavity field [1–5]. Different from natural atoms, the properties of artificial atoms can be artificially designed and controlled. Recently, the achieved new progresses in circuit-QED study open up a new prospect for quantum states preparing and quantum information processing [6–8].

It is well known that the entanglement between qubits should be maintained as they dynamically evolve in time for many applications of interest [9–11]. Quantum entanglement has become an important physical resource for quantum communication and information processing like universal geometric quantum gates [12], quantum teleportation [13], superdense coding [14], quantum key distribution and telecoloning [15].

By coupling a big Josephson junction with two superconducting charge qubits, implementing control of amplitude and frequency modulation via external microwave field, Ref. [16] realized controllable manipulation of single qubit or two-qubit. For the convenience of regulating, using the big Josephson junction as data bus and coupling the single qubits with it, then there will be interactions between two qubits of large distance, which is a universal and integrated method for quantum information processing based on the solid system cavity QED. Apparently, the influence of quantum property of microwave field on the entanglement dynamics in circuit cavity is crucial. Besides, the controllable coupling, measuring for the qubit state is also a very important step in quantum information processing. Undoubtedly, the effects of microwave field on the quantum state of superconducting qubits can display via the output current flowing through the qubits.

The entanglement exchange of cavity field with qubits and the influence of interaction effects on quantum entanglement dynamics are the hot issues people investigating. In the present work, we use the circuit cavity model, which is constructed by coupling two superconducting charge qubits via a big Josephson junction, to investigate the entanglement and supercurrent dynamical features of coupled superconducting qubits when the cavity field is respectively in coherent state, even coherent state and odd coherent state.

2. Model

We model a big Josephson junction as data bus and couple two superconducting charge qubits through it, which is described in Ref. [17]. When the dc biased magnetic field Φ_e is externally applied, the Hamiltonian for the system is given by

$$H = \sum_{k=1}^{2} \left[E_k(V_{xk}) - 3E_{Jk} \cos\left(\frac{\pi \Phi_e}{\Phi_0} - \frac{\gamma}{2} \right) \cos\varphi_k \right] - E_{J0} \cos\gamma \qquad (1)$$

$E_k(V_{xk}) = E_{Ck}(n_k - C_k V_{xk}/2e)^2$ is the electrostatic energy of charge qubits, $E_{Ck} = 2e^2/(C_k + 2C_{Jk})$ and E_{Jk} is the charging energy and Josephson energy of kth qubit, and E_{J0} is the Josephson energy

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of the big junction. Exerting a time-variant microwave field, the manipulation for both single qubit and coupling qubits could be achieved through controlling amplitude modulation or frequency modulation. We focus on how the microwave fields affect entanglement and superconducting of coupling qubits, where the superconducting qubits are placed inside microwave field.

Considering the situation of quantization microwave field $\Phi_q a^+ + \Phi_q^* a$, Φ_q is the amplitude and $a^+(a)$ is the creation and annihilation operator.

After some calculations, the interactive Hamiltonian between quantized magnetic field and two superconducting charge qubits can be written as

$$H_I = g(a^+\sigma_1^-\sigma_2^- + a\sigma_1^+\sigma_2^+) \tag{2}$$

$$g = -\frac{\Phi_q I_{C1} I_{C2}}{I_0}\sin\left(\frac{2\pi\Phi_e}{\Phi_0}\right) \tag{3}$$

I_0 and I_{ck} are the critical current of the large junction and kth superconducting qubit. We suppose that the two superconducting qubits is in the Bell state with spins anti-correlated,

$$\left|\psi_Q(0)\right\rangle = \cos\theta\left|gg\right\rangle + e^{i\varphi}\sin\theta\left|ee\right\rangle \tag{4}$$

and the cavity field is initially in the position state of number states

$$\left|\psi_F(0)\right\rangle = \sum_n f(n)\left|n\right\rangle \tag{5}$$

$f(n)$ is the probability amplitude of photon number state distribution of quantum field. So the initial state of the system can be written as $\left|\psi(0)\right\rangle = \left|\psi_F(0)\right\rangle \otimes \left|\psi_Q(0)\right\rangle$. In the interaction picture, the vector of the system at any time t is written as [18]

$$\begin{aligned}\left|\psi(t)\right\rangle &= \sum_{n=0}^{\infty}\left[x_n(t)\left|e,\ e,\ n\right\rangle + y_n(t)\left|g,\ g,\ n+1\right\rangle\right]\\ &\quad + f(0)\cos\theta\left|g,\ g,\ 0\right\rangle\end{aligned} \tag{6}$$

where

$$x_n(t) = e^{i\varphi}f(n)\sin\theta\cos\left(\sqrt{n+1}gt\right) - f(n+1)\cos\theta\sin\left(\sqrt{n+1}gt\right),$$

$$y_n(t) = e^{i\varphi}f(n)\sin\theta\sin\left(\sqrt{n+1}gt\right) + f(n+1)\cos\theta\cos\left(\sqrt{n+1}gt\right).$$

After knowing the state vector of circuit cavity, we can research the physical properties of superconducting coupling qubits and quantum field. In the process of quantum information transmission, the key problem is how to accurately read out the output quantum state. To implement the readout of two-qubit states, we need to calculate the circulating supercurrent $\hat{I}$ contributed by the two qubits. The operator of the supercurrent $\hat{I}$ of the two qubits is given by

$$\begin{aligned}\hat{I} &= \sin\left(\frac{\pi\Phi_e}{\Phi_0}\right)\left(I_{c1}\sigma_1^x + I_{c2}\sigma_2^x\right)\\ &\quad - \frac{1}{4I_0}\sin\left(\frac{2\pi\Phi_e}{\Phi_0}\right)\left(I_{c1}^2 + I_{c2}^2 + 2I_{c1}I_{c2}\sigma_1^x\sigma_2^x\right)\end{aligned} \tag{7}$$

In the quantum vector $\left|\psi(t)\right\rangle$, the supercurrent can be obtained by

$$\begin{aligned}(t) &= -\frac{1}{4I_0}\sin\left(\frac{2\pi\Phi_e}{\Phi_0}\right)\\ &\quad\times\left\{I_{c1}^2 + I_{c2}^2 + 4I_{c1}I_{c2}Re\left[f(0)a_{-1}(gt)\cos\theta + \sum_{n=0}^{\infty}a_n(gt)b_n(gt)\right]\right\}\end{aligned} \tag{8}$$

where

$$\begin{aligned}a_n(t) &= e^{-i\varphi}f^*(n+1)\sin\theta\cos\left(\sqrt{n+2}gt\right)\\ &\quad - f^*(n+2)\cos\theta\sin\left(\sqrt{n+2}gt\right),\end{aligned}$$

$$b_n(t) = e^{i\varphi}f(n)\sin\theta\sin\left(\sqrt{n+1}gt\right) + f(n+1)\cos\theta\cos\left(\sqrt{n+1}gt\right).$$

Eq. (8) shows that, if the dc biased magnetic field Φe is constant, the supercurrent consists of dc component and time-variant component. The time-variant component relies on the quantum state of circuit cavity and the photon number distribution of quantum field. Thus, we can obtain the quantum property of quantum field through experimentally measuring the output current of superconducting qubits.

3. Discussions

Entanglement plays a key role in quantum computation and quantum information, which is a kind of nonclassical correlation that is not available in classical world. Quantum information transmission relies on the entanglement property between quantum bits. We adopt the concurrence defined by Wootters [19] to measure the system entanglement. One can calculate that the concurrence for the initial states $\left|\psi\right\rangle$ is

$$C(t) = 2\left|\sum_{n=0}^{\infty}x_{n+1}(t)y_n^*(t) + x_0(t)f^*(0)\cos\theta\right| \tag{9}$$

Obviously, same as the output current of qubit, concurrence also relies on the quantum state of circuit cavity and the photon number distribution of quantum field. It factually reflects that in the same quantum system, there's a certain correlation between concurrence and supercurrent. In this section, we compare the time-evolving dynamics of concurrence and supercurrent in virtue of numerical method.

For convenience, the two qubits are supposed to have identical critical current $I_{c1} = I_{c2} = I_c$, and we define a reduced supercurrent as

$$\tilde{I}(t) = 1 + 2Re\left[f(0)a_{-1}(gt)\cos\theta + \sum_{n=0}^{\infty}a_n(gt)b_n(gt)\right] \tag{10}$$

We discuss the quantum correlation dynamics of the qubits when the field is in the general coherence state

$$\left|\psi_F(0)\right\rangle = e^{-|\alpha|}\sum_n\frac{1+(-1)^n r}{\sqrt{1+r^2+2re^{-2|\alpha|^2}}}\frac{\alpha^n}{\sqrt{n!}}\left|n\right\rangle \tag{11}$$

$\alpha = \sqrt{\bar{n}}e^{i\phi}$. $\bar{n}$ is the average number of photons of the field, parameter r is 0, −1, 1, corresponding to coherent state, odd coherent state and even coherent state respectively. Subsequently, we will investigate the effect from the relative phase of the coupling qubits and the average number of photons.

3.1. Effect of the relative phase

We consider the case that the two qubits are initially in $\left(\left|gg\right\rangle + e^{i\varphi}\left|ee\right\rangle\right)/\sqrt{2}$, but the field is initially in different coherent state.

(i) Coherent state

When the field is initially in the typical coherent state, the time evolution of concurrence and supercurrent is respectively