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Dark optical solitons and conservation laws for parabolic and dual-power law nonlinearities in (2 + 1)-dimensions

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ABSTRACT

This paper studies the dynamics of optical solitons with parabolic and dual-power law nonlinearities. The dark 1-soliton solution is first obtained by the ansatz method along with the necessary constraint conditions, for both of these nonlinearities. Subsequently, the invariance, conservation laws and double reductions of the governing nonlinear Schrödinger's equation are studied and the conserved densities are thus revealed.

1. Introduction

The theory of solitons is a very important area of research in the field of applied physics [1–15]. In particlar, optical solitons play a vital role in daily lives. The information carrying capacity of optical fibers, for transcontinental and transoceanic distances, is achieved by the aid of optical solitons. There have been a lot of research activities in this area. A plethora of papers has been published in this field for the past few decades. There is still a long way to go, in this direction. One of the main focus in this area is the integrability of the governing nonlinear Schrödinger's equation (NLSE). It is always a challenging feature to address the integrability aspects of NLSE in (2 + 1)-dimensions. While the Kerr law and power law are already studied in this context, it is the parabolic and dual-power laws that still deserves attention [11,12]. There are still a lot of open problems in this area as of today. This paper will thus fill in a small gap in this context. It is the issue of dark solitons that will be covered, in this paper.

This paper will thus study the dark optical solitons for parabolic and dual-power laws of nonlinearities in (2 + 1)-dimensions. The ansatz method will be applied to obtain the exact 1-soliton solution to this equation along with a couple of constraint conditions that are needed in order for these dark solitons to exist. Subsequently, this paper will address the conservation laws of the NLSE with parabolic and dual-power laws of nonlinearity. The invariance and the double reduction methods will be adopted to retrieve the conserved densities. The commutator table will also be displayed.

2. Governing equation

The dynamics of soliton propagation in (2 + 1)-dimensions in non-Kerr law media is governed by the NLSE that is given by [1,2]

$$iq_t + a(q_{xx} + q_{yy}) + F(|q|)^2 q = 0 \tag{1}$$

Here in (1), the first term represents the evolution term, while the coefficients of a are the group velocity dispersion (GVD) terms in x- and y-directions respectively. Finally, the functional F represents the non-Kerr law nonlinearity, in general. The particular case, when $F(s) = s$, (1) reduces to Kerr law nonlinearity. The solitons are the outcome of a delicate balance between GVD and nonlinearity. The complex variable $q(x, y, t)$ is the wave profile where x and y are the spatial variables while t represents the temporal variable. This paper will study the dark optical solitons for two forms of the functional F. They are described in the next two subsections.

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2.1. Parabolic law nonlinearity

For parabolic law nonlinearity, the functional F is given by [1]

$$F(s) = b_1 s + b_2 s^2$$

where b_1 and b_2 are constants. This kind of nonlinearity is due to the presence of substantial $\chi^{(5)}$ nonlinearity, also known as fifth order susceptibility. This is typically observed in transparent glass with intense femtosecond pulses at 620 nm [9–11]. Therefore NLSE in (2 + 1)-D with parabolic law nonlinearity is [9]

$$iq_t + a(q_{xx} + q_{yy}) + (b_1|q|^2 + b_2|q|^4)q = 0 \tag{2}$$

where a, b_1 and b_2 are constants.

To determine the topological 1-soliton solution of the NLSE (1) explicitly, we adopt a soliton ansatz of the type [15]

$$q(x, t) = (A + B\tanh\tau)^p e^{i\phi} \tag{3}$$

where

$$\tau = B_1 x + B_2 y - vt \tag{4}$$

$$\phi = -\kappa_1 x - \kappa_2 y + \omega t + \theta \tag{5}$$

where in (2)–(4) A, B, B_1 and B_2 are free parameters and v is the velocity of the wave. κ_1 and κ_2 are the soliton frequencies in the x- and y-directions, while ω is the wave number of the soliton and θ is the phase constant. Also, the unknown exponent p will be determined during the course of the derivation of the soliton solution to (1). Substituting (3) into (2) and then decomposing into real and imaginary parts give

$$\begin{aligned}
&-\omega(A + B\tanh\tau)^p + aB^2(B_1^2 + B_2^2)p(p-1)\left(1 - \frac{A^2}{B^2}\right)^2(A + B\tanh\tau)^{p-2}\\
&+\frac{ap(p+1)(B_1^2 + B_2^2)}{B^2}(A + B\tanh\tau)^{p+2} + 2ap(2p-1)\left(1 - \frac{A^2}{B^2}\right)A(B_1^2 + B_2^2)(A + B\tanh\tau)^{p-1}\\
&+a\left[2(B_1^2 + B_2^2)p^2\left(\frac{3A^2}{B^2} - 1\right) - \kappa_1^2 - \kappa_2^2\right](A + B\tanh\tau)^p\\
&-\frac{2ap(2p+1)A(B_1^2 + B_2^2)}{B^2}(A + B\tanh\tau)^{p+1} + b_1(A + B\tanh\tau)^{3p} + b_2(A + B\tanh\tau)^{5p}
\end{aligned} \tag{6}$$

and

$$\begin{aligned}
&-pBv\left(1 - \frac{A^2}{B^2}\right)(A + B\tanh\tau)^{p-1} - \frac{2pvA}{B}(A + B\tanh\tau)^p + \frac{pv}{B}(A + B\tanh\tau)^{p+1}\\
&-aB(\kappa_1 B_1 + \kappa_2 B_2)2ap\left(1 - \frac{A^2}{B^2}\right)(A + B\tanh\tau)^{p-1} - \frac{4apA(\kappa_1 B_1 + \kappa_2 B_2)}{B}(A + B\tanh\tau)^p\\
&+\frac{2ap}{B}(\kappa_1 B_1 + \kappa_2 B_2)(A + B\tanh\tau)^{p+1} = 0
\end{aligned} \tag{7}$$

respectively. By equating the exponents of $(A + B\tanh\tau)^{p+2}$ and $(A + B\tanh\tau)^{5p}$ functions in (6), one gets

$$p + 2 = 5p \tag{8}$$

That gives the following value of p:

$$p = \frac{1}{2} \tag{9}$$

It needs to be noted that the same results are obtained when the exponents $p + 1$ and $3p$ are equated in (6).

Now from (7), setting the coefficients of the linearly independent functions $(A + B\tanh\tau)^{p+j}$ to zero, where $j = 0, \pm 1$, gives

$$A = B \tag{10}$$

$$v = -2a(\kappa_1 B_1 + \kappa_2 B_2) \tag{11}$$

Also from (6), setting the coefficients of the linearly independent functions $(A + B\tanh\tau)^{p+j}$ to zero, where $j = 0, \pm 1, \pm 2$, gives

$$\omega = a[(B_1^2 + B_2^2) - \kappa_1^2 - \kappa_2^2] \tag{12}$$

$$A = B \tag{13}$$