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Analysis of the temporal power spectral models of angel of arrival fluctuations for optical waves propagating through weak non-Kolmogorov turbulence

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ABSTRACT

New analytical expressions for the temporal power spectral models of angle of arrival (AOA) fluctuations are derived for optical plane and spherical waves propagating through weak non-Kolmogorov turbulence. They consider the finite turbulence inner and outer scales, and have a general power law value in the range of 3–4 instead of the standard power law value of 11/3. The results derived in this work can reduce correctly to the previously published analytic expressions for the case of plane and spherical waves propagation through Kolmogorov turbulence case. These results are useful for the understanding the potential impact of derivations from the standard Kolmogorov spectrum.

1. Introduction

Atmospheric turbulence produces a series of effects (including AOA fluctuations, irradiance scintillation, beam spread, and so on) on the imaging or laser systems. AOA fluctuations of optical waves in the plane of receiver aperture are related to image dancing in the focal plane of an imaging or laser system. Atmospheric turbulence is a space and time-varying random medium. To simplify the complexity, Taylor's frozen hypothesis [1] is used to investigate the temporal power spectrum of AOA fluctuations for optical waves propagating through atmospheric turbulence. The hypothesis assumes that the large-scale mean motions (involving the motion of entire spectrum of turbulence scales) caused by the wind predominates the temporal variation of atmospheric turbulence. With this hypothesis, the temporal power spectrum of atmospheric turbulence can be determined directly from the spatial covariance function of atmospheric turbulence with known wind velocity perpendicular to optical waves propagating path.

In recent years, many researchers have focused on non-Kolmogorov turbulence case. Compared with Kolmogorov turbulence, non-Kolmogorov turbulence covers a more wide range of atmospheric layers and this has been demonstrated by experimental results [2–5] and theoretical investigations [6,7]. Non-Kolmogorov atmospheric turbulence spectral model has been used to investigate the temporal power spectrum of AOA fluctuations for plane and spherical waves propagating through weak non-Kolmogorov turbulence [8]. However, they have not considered the influences of turbulence inner and outer scales yet. Then, a series of atmospheric turbulence spectral models which consider the finite turbulence inner and outer scales are proposed for the non-Kolmogorov turbulence, including the generalized von Karman spectrum [9], the generalized modified atmospheric spectrum [10], and the generalized exponential spectrum [11]. In comparison, the generalized modified atmospheric spectral model is the only physical turbulence refractive spectral model which can fit with the experimental data. Therefore, in this study, this spectral model is adopted to derive the theoretical expressions of the temporal power spectral modes of AOA fluctuations for plane and spherical waves propagating through weak non-Kolmogorov turbulence. Then, the influences of finite turbulence outer scale, spectral power law, and diameter of the aperture receiver on the temporal power spectrum of AOA fluctuations are analyzed.

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2. Generalized modified spectral model for non-Kolmogorov turbulence

The generalized modified spectral model [10] can be applied in non-Kolmogorov atmospheric turbulence, which considers finite turbulence inner and outer scales and has a general spectral power law value in the range of 3–5 instead of standard power law value 11/3 for Kolmogorov turbulence. Specifically, this spectral model has the following form:

$$\Phi_n(\kappa,\alpha,l_0,L_0)=A(\alpha)\cdot\hat{C}_n^2\cdot\kappa^{-\alpha}\cdot f(k,l_0,L_0,\alpha)\quad(0\le\kappa<\infty,\quad 3<\alpha<5), \tag{1}$$

$$f(\kappa,l_0,L_0,\alpha)=\left[1-\exp\left(-\frac{\kappa^2}{\kappa_0^2}\right)\right]\left[1+a_1\left(\frac{\kappa}{\kappa_l}\right)-b_1\left(\frac{\kappa}{\kappa_l}\right)^{\beta_1}\right]\cdot\exp\left(-\frac{\kappa^2}{\kappa_l^2}\right). \tag{2}$$

where $\hat{C}_n^2$ is the generalized refractive-index structure parameter with units $m^{3-\alpha}$, k denotes the magnitude of the spatial-frequency vector with units of *rad/m* and is related to the size of turbulence cells, $f(k,l_0,L_0,\alpha)$ describes the influence of finite turbulence inner and outer scales, $\kappa_l=c(\alpha)/l_0$, $\kappa_0=C_0/L_0$, l_0 and L_0 are the turbulence inner and outer scales, respectively. The choice of C_0 depends on the specific application, in this study, it is set to 4τ just as [12]. The coefficients of a_1, b_1 and β_1 in the generalized modified atmospheric spectrum depend on experimental results [10]. $A(\alpha)$ and $C(\alpha)$are given by [10]:

$$A(\alpha)=\frac{\Gamma(\alpha-1)}{4\pi^2}\sin\left[(\alpha-3)\frac{\pi}{2}\right],\quad c(\alpha)=\left\{\pi A(\alpha)\left[\Gamma\left(-\frac{\alpha}{2}+\frac{3}{2}\right)\left(\frac{3-\alpha}{3}\right)+a_1\cdot\Gamma\left(-\frac{\alpha}{2}+2\right)\left(\frac{4-\alpha}{3}\right)-b_1\cdot\Gamma\left(\frac{-\alpha+3+\beta_1}{2}\right)\left(\frac{3+\beta_1-\alpha}{3}\right)\right]\right\}^{1/\alpha-5}. \tag{3}$$

when $\alpha=11/3$, A(11/3) = 0.33 and $c(11/3)\approx3.3$, Eq. (1) is reduced to the Kolmogorov turbulent exponential spectral model. And when $l_0\to0$, $L_0\to0$, Eq. (1) becomes the general non-Kolmogorov spectral model [13]:

$$\Phi_n(\kappa,\alpha)=A(\alpha)\cdot\hat{C}_n^2\cdot\kappa^{-\alpha}\quad(0\le\kappa<\infty,\quad 3<\alpha<5). \tag{4}$$

3. Temporal power spectral model of AOA fluctuations in weak non-Kolmogorov turbulence

Following Tatarski [1], the temporal power spectrum of AOA fluctuations $W_\theta(\omega,\beta)$ is the Fourier transform of the temporal covariance function of AOA $C_\theta(t,\beta)$ [1]:

$$W_\theta(\omega,\beta)=4\int_0^\infty C_\theta(t,\beta)\cos(\omega t)dt. \tag{5}$$

With Taylor frozen turbulence hypothesis, $C_\theta(t,\beta)$ can be determined from the spatial covariance function of AOA $C_\theta(\rho,\beta)$. And $C_\theta(\rho,\beta)$ takes the form as [14]:

$$C_\theta(\rho,\beta)=\pi k^{-2}\int_0^\infty\kappa^3W_\phi(\kappa)G_D(\kappa)\left[J_0(\rho\kappa)-\cos(2\beta)J_2(\rho\kappa)\right]d\kappa, \tag{6}$$

where ρ represents the geometrical separation between points in the plane transverse to the direction of propagation, β is the angle between the baseline and AOA observation axis, $k=2\pi/\lambda$ and λ denotes the optical wavelength. $J_0(\rho k)$ and $J_2(\rho k)$ denote the zero and second order Bessel functions, respectively. Taylor frozen turbulence hypothesis make the association of $\rho=\nu_\perp t$ satisfied, where $\nu_\perp$ denotes the wind velocity perpendicular to optical waves propagating path. In this case, $C_\theta(t,\beta)$ can be written as:

$$C_\theta(t,\beta)=\pi k^{-2}\int_0^\infty\kappa^3W_\phi(\kappa)G_D(\kappa)\left[J_0(\nu_\perp t\kappa)-\cos(2\beta)J_2(\nu_\perp t\kappa)\right]d\kappa, \tag{7}$$

$W_\phi(k)$ is the wave-front phase power spectrum. $G_D(k)$ denotes the point-spread function of the receiver aperture [17]:

$$G_D(\kappa)=\exp\left[-\frac{c^2D^2\kappa^2}{4}\right],\quad c=0.52. \tag{8}$$

For plane and spherical waves, $W_\phi(k)$ takes different expressions [15,16]:

$$W_{\phi(pl)}(\kappa)=2\pi k^2\int_0^L\Phi_n(\kappa)\cos^2\left(\frac{\kappa^2z}{2k}\right)dz, \tag{9}$$

$$W_{\phi(sp)}(\kappa)=2\pi k^2\int_0^L\Phi_n(\kappa)\left(\frac{z}{L}\right)^2\cos^2\left[\frac{\kappa^2z(L-z)}{2kL}\right]dz. \tag{10}$$

$W_\phi(pl)(k)$ and $W_\phi(sp)(k)$ are the wave-front phase power spectrum for plane and spherical waves, respectively.