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Non singular material parameters for arbitrarily elliptical–cylindrical invisibility cloaks

Vahid Gholizadeh [a,*], Ali Rostami [a,b], Saeed Golmohammadi [a], Karim Abbasian [a], Alisa Javadi [a]

[a] Nanophotonics Group, Department of Engineering-Emerging Technologies, University of Tabriz, Tabriz 5166614761, Iran
[b] Faculty of Electrical and Computer Engineering, University of Tabriz, Tabriz 5166614761, Iran

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ABSTRACT

Simple general expressions for an important class of invisibility cloaks with less radial symmetry are derived by using transformation optics theory. Material parameters of arbitrarily elliptical–cylindrical invisibility cloak are obtained using a transformation in the elliptical–cylindrical coordinate system. It is explicitly shown that the azimuthal terms of material parameters in ideal case, like circular-cylindrical cloaks, are infinite at the whole interior surface of the cloak which makes ideal cloak impractical. Maxwell's equations are obtained in arbitrarily elliptical–cylindrical coordinate system and then the reduced material approximation is expanded to eliminate such singularities in the cylindrical cloaks with arbitrarily elliptical cross sections. The impedance of the cloak is derived and the cloak with reduced material approximation is designed so that the cloak's impedance at outer boundary experiences no change. An approximated 8 layer practical cloak is proposed such that each layer is divided to 32 slices azimuthally. Functionality of the proposed cloaks are numerically confirmed by two-dimensional finite element simulations where we illustrated that both layered and reduced cloaks with no singularities in their material parameters and therefore much easier realization possibility, show very good cloaking performance.

1. Introduction

The exciting issue of invisibility cloaks has received much attention in recent years [1–22]. Firstly, an invisibility cloak was proposed by Pendry et al. [1], which based on Maxwell's equations invariance characteristics, can exclude electromagnetic fields from a transparent object without perturbing the exterior fields, thus rendering the interior effectively "*invisible*" to the outside and protects object inside the cloak from detection. A conformal mapping approach has been used to design a medium that can make invisibility cloak working in a short-wavelength geometrical limit and two-dimension space [2,3]. Shortly, the cloaking effect has also been proved by ray tracing method, assuming geometrical optics limit [4]. The experimental example of cylindrical invisibility cloak at microwave frequencies has been realized by metamaterials [5].

More specific studies have been done such as: fully electromagnetic simulations of the circular-cylindrical version of cloaking structures [6], designing nonmagnetic cylindrical cloak working in optical frequencies with minimized scattering [7,8], broadband cylindrical invisibility cloaks [9], investigation of scattering characteristics and improvement in material parameters of simplified cylindrical invisibility cloaks [10,11]. Among all the researches, cloaks with radially symmetric spherical and cylindrical geometries have received much attention. In principle, objects of any shape can be hidden [1–22]. So, two-dimensional cylindrical cloaks of an arbitrary cross section have been proposed [12–16], and elliptical–cylindrical cloaks have also been studied [17–22].

The most important problem in theoretical researches on cloaking is how to construct the permittivity and permeability tensors of cloak shell materials. In order to design practical cloaks, the tensors should be as simple as possible. According to the transformation optics theory stated in [1], to build material parameter tensors of the cloak shell, one should first choose a new curvilinear coordinate system, by which the spatial distortion associated with the cloak's shape can be described and the coordinate transformation between the original Cartesian and the new coordinate system can be performed, correctly.

In order to obtain material parameter tensors of the elliptical–cylindrical invisibility cloak, there have been two points of view. One method is using the circular-cylindrical system of coordinates to obtain material parameters of arbitrarily elliptical–cylindrical cloak, in which the cloaked objects are shrunk to a point [16]. This method has the advantage that it is valid for cylindrical objects of both circular and elliptical cross-sections, however it has the disadvantage that the material parameters are

* Corresponding author.
E-mail address: v.gholizadeh87@ms.tabrizu.ac.ir (V. Gholizadeh).

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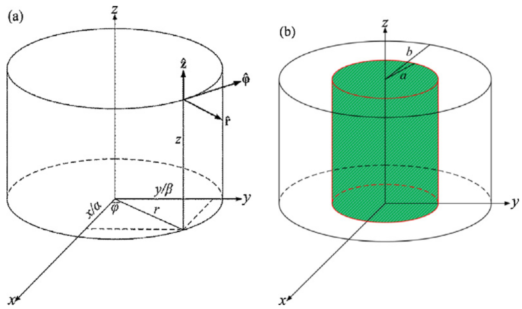

Fig. 1. Coordinate system for the elliptical–cylindrical annular cloak. (a) The elliptical–cylindrical coordinate system (r, φ, z) and the associated unit vectors. (b) The elliptical–cylindrical coordinate system after transformation. The shaded region is completely excluded from the system. a and b denote the inner and the outer boundary of cloak shell, respectively.

infinite on the inner boundary of the cloak. The other method is considering the orthogonal hyperbolic–elliptic system of coordinates, in which the cloak is obtained by blowing up a line segment (focus length) to an ellipse [19–22]. This method solves the problem of singularities on the inner boundary of the cloak, but it is only valid for cloaks with elliptical cross sections. In addition the cloak is incident-angle dependent in this case and the cloaking effect can be considered only in nearly circular cases [19] and it has been shown that this cloak is inherently visible even in its ideal case and the cloaking effect can be valid in the presence of a perfect electric conductor (PEC) [21,22].

To ease experimental realization at microwave frequencies, Schurig et al. have simplified the material parameters of circular-cylindrical cloak such that only one component of material parameters is spatially varying and the necessity of infinite azimuthal constant is terminated [5]. It has been shown that the best type of simplified circular-cylindrical cloak is one which is impedance matched to the free space at the outer interface and therefore, has zero reflectance [8,11].

In this paper, we focus on arbitrarily elliptical–cylindrical invisibility cloak which is obtained by blowing up a point to an ellipse. We use the transformation optics method of Ref. [1] to obtain material parameters of the ideal cloak in the arbitrarily elliptical–cylindrical coordinate system. We show that the material parameters in elliptical cloaking are singular at the whole inner boundary of the cloak. To get practical elliptical cloaks material parameters, we should eliminate the singularities and use approximated cloaks with good cloaking performance. In this paper, we obtain two of such approximated elliptical cloaks, reduced material and layered one. We utilize "differential geometry" to obtain Maxwell's equations in arbitrarily elliptical–cylindrical coordinate system [23,24], and consequently, extend the reduced material approximation to cylindrical cloaks with elliptical cross-sections. To achieve the best cloaking performance in reduced material approximation, we obtain cloak impedance and choose reduced material parameters in such a way that the impedance of simplified cloak remains the same as that of the ideal one. The obtained cloak with simplified material parameters can be realized by metamaterials. In addition, we propose an eight layer approximation of the ideal cloak that each layer has divisions of $\pi/16$ rad azimuthally. Numerical two-dimensional finite element simulations are done to confirm functionality of the proposed cloak.

2. Ideal cloak

According to the transformation optics theory [1], the starting point in design of an invisibility cloak is choosing a new curvilinear coordinate system, by which the spatial distortion associated with the cloak's shape can be described and the cloak material parameters can be obtained by a coordinate transformation between the original Cartesian and the new coordinate systems. Due to the elliptical shape of the cloak, we select elliptical–cylindrical coordinate system (r, φ, z) as the new coordinate. For an ellipse with its semi-axes laying along x-, and y-directions with lengths αr and βr, respectively, the ellipse is expressed by

$$\left(\frac{x}{\alpha r}\right)^2 + \left(\frac{y}{\beta r}\right)^2 = 1, \tag{1}$$

where $\alpha, \beta \geq 1$ are positive numbers that describe the aspect ratio of the ellipse and r is the scaled distance between an arbitrary point on the ellipse and origin. For an elliptical–cylindrical annular cloak, we first define a coordinate system $x^{i} = (r, \varphi, z)$ in terms of Cartesian coordinates $x^{i} = (x, y, z)$, with $i, i = 1, 2, 3$, via relationships

$$x^{i'} = \begin{cases} r = \sqrt{\left(\dfrac{x}{\alpha}\right)^2 + \left(\dfrac{y}{\beta}\right)^2} \\ \varphi = \tan^{-1}\left(\dfrac{\alpha y}{\beta x}\right) \\ z = z \end{cases}, \quad i' = 1, 2, 3. \tag{2}$$

The resulting elliptical–cylindrical coordinate system (r, φ, z) is illustrated in Fig. 1(a). This coordinate system is a generalization of conventional circular-cylindrical coordinate system. The constant-r contours in the x–y plane represent a family of ellipses with a constant axial ratio β/α, while the constant-φ contours are a collection of radial lines. This system is different from the conventional hyperbolic–elliptic coordinate system [19–22]. Additionally, (r, φ, z) does not construct an orthogonal coordinate system in general, the orthogonal case is obtained when the axial ratio $\beta/\alpha = 1$ where the ellipse reduces to the circle, i.e. the conventional circular-cylindrical coordinate system is obtained. The transformed coordinate system is illustrated in Fig. 1(b), where the elliptic region (indicated by the shaded area) is excluded from the system, completely.

We choose a spatial transformation that maps the whole elliptical–cylindrical domain $0 \leq r' \leq b$ in virtual (before transformation) space, to the elliptical–cylindrical shell $a \leq r \leq b$ in physical (after transformation) space. a and b are inner and outer shell coordinate parameters of the elliptical cylindrical cloak, respectively. As Fig. 1(b) illustrates, physical regions with $r < a$ are not reached by electromagnetic waves and for $r > b$ the virtual space coordinates match with the coordinates of physical space. The mapping function is given by [1]

$$\begin{cases} r = f_{(r')} \\ \varphi = \varphi' , \quad 0 \leq \varphi \leq 2\pi \\ z = z' \end{cases} \tag{3}$$

According to the method stated in Ref. [24], we can deduce relative permittivity and permeability tensors of the elliptical–cylindrical cloak as

$$\varepsilon_{rr} = \mu_{rr} = \frac{r'R}{r} + \left(\frac{r'R}{r} - 1\right) e^2 \sin^2\varphi \cos^2\varphi, \tag{4}$$

$$\varepsilon_{r\varphi} = \mu_{r\varphi} = (r - r'R)\left(\frac{\beta}{\alpha}\cos^2\varphi + \frac{\alpha}{\beta}\sin^2\varphi\right) e \sin\varphi \cos\varphi, \tag{5}$$

$$\varepsilon_{\varphi r} = \mu_{\varphi r} = \left(\frac{1}{r} - \frac{1}{r'R}\right)\left(\frac{\alpha}{\beta}\cos^2\varphi + \frac{\beta}{\alpha}\sin^2\varphi\right) e \sin\varphi \cos\varphi, \tag{6}$$

$$\varepsilon_{\varphi\varphi} = \mu_{\varphi\varphi} = \frac{r}{r'R} + \left(\frac{r}{r'R} - 1\right) e^2 \sin^2\varphi \cos^2\varphi, \tag{7}$$

$$\varepsilon_{zz} = \mu_{zz} = \frac{r'}{rR}, \tag{8}$$