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Azimuthally modulated ring solitons in Bessel photonic lattices

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ABSTRACT

Energy spectrum, stability and internal modes of azimuthally modulated ring solitons are investigated in defocusing photorefractive crystal with imprinted Bessel photonic lattices. It is shown that internal modes of the azimuthally modulated ring solitons may have both real parts and imaginary parts. If there exists internal modes of the stable azimuthally modulated ring solitons, the solitons perform long-distance and quasi-periodic oscillation of intensity and shape under the perturbation of internal modes.

1. Introduction

Wave propagation in optical periodic structures has been a focused topic in optics and Bose–Einstein condensates and has attracted great attention in recent years. The introduction of optical lattices into nonlinear medium provides an effective way to guide or control the dynamics of solitons. The basic features of wave propagation in transversely periodic optical lattice (modulation of refractive index) are fundamentally different from those in uniform media. In these periodic nonlinear lattices, the self-focusing or defocusing characteristic may balance the lattice diffraction, resulting in a lattice solitons or discrete solitons [1–4]. Discrete solitons exhibit a number of unique properties including controllable steering and switching that makes them promising to demonstrate all-optical routing concepts. The most important type of periodic lattice is square lattice or pixel lattice [5,6]. Because of the equivalence between optical waves in a periodic dielectric structure and electrons in a periodic atomic potential, the characteristics of wave propagating in square lattices can be described in terms of the concepts of Brillouin zone and band gap structure [7,8]. The periodicity divides the linear transmission spectrum of waves in such square structures into bands of propagating modes or Bloch modes separated by forbidden gaps. Bessel lattices, another kind of quasi-periodic concentric photonic lattices have also attracted much interest [9–14]. As eigenmode solutions of linear Schrödinger equation, Bessel beams can propagate in the free space without diffraction, and Bessel lattices can be induced by a number of ways, including illumination through a narrow annular slit placed in the focal plane of a lens or axicon [15] or computer-generated hologram [16]. Recently, Fischer et al. [17] generated higher-order azimuthally modulated Bessel optical lattices in photorefractive crystals by employing a phase-imprinting technique.

Kartashov et al. investigated multipole-mode solitons [11–13] and ring-profile vortex solitons [9] in Bessel optical lattices imprinted in focusing and defocusing Kerr nonlinear media. They found that stable multi-pole solitons existed in Bessel lattices when the propagation constant exceeded the critical value; in a high-power limit the multi-pole solitons featured a multi-ring structure. There are some issues that should be emphasized about these interesting papers. The nonlinearity of the medium is cubic Kerr effects, the initial input beam is infinitely wide, and the internal modes of the solitons have not been revealed yet. In a nonintegrable model, a small perturbation may create an internal oscillation of a solitary wave. Such oscillations introduce qualitatively new features into the system dynamics responsible for long-living of the solitary wave shape and resonant solitons interaction and they are usually relative to the so-called internal modes or shape modes of solitons [18–22]. Internal modes have been analyzed for the so-called kink solitons, topological solitary waves of the Klein–Gordon-type models [20–22], sine-Gordon solitons [23], fundamental solitons in saturable nonlinear media [24], and vortex solitons in cubic-quintic nonlinear media [25]. All these investigations on internal modes are restricted to one-dimensional (1-D) solitons or 2-D azimuth-independent solitons in homogeneous material. Furthermore, no study on internal modes of solitons in photorefractive media is found. The investigation of internal modes of multi-pole solitons in photonic lattices remains a challenge.

To further investigate this topic in this paper, we study 2-D azimuthally modulated ring solitons in defocusing photorefractive media in which concentric Bessel optical lattices are imprinted,

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taking 14-poled solitons for example. Such a soliton is confined in a single ring of the Bessel lattice, and is azimuthally modulated so that they look like a necklace. The emphases of the work focus on the perturbation eigenvalue problem, by which the stability, internal modes of the necklace-like solitons guided by Bessel lattices imprinted in defocusing photorefractive media, will be investigated in detail. The roles that internal modes play will also be studied by observing the propagation of the necklace solitons under perturbation of the internal modes. We will find that the internal modes of azimuthally modulated ring solitons in Bessel optical lattices have both real part and imaginary part, which differs from the one-dimensional cases [20]. The solitons perform quasi-periodic long-distance oscillation of intensity and shape under the perturbation of internal modes.

2. Energy spectrum of azimuthally modulated ring solitons in Bessel Lattices

Assume that a permanent Bessel lattice is imprinted in a photorefractive defocusing medium by the method of photoinduction or fabrication and such index lattice is not affected by other light wave, then the dynamics of a signal beam propagating through the Bessel lattice is modeled by the normalized nonlinear Schrödinger equation for the complex field amplitude q [26]:

$$i\frac{\partial q}{\partial z}+\frac{1}{2}\left(\frac{\partial^2 q}{\partial x^2}+\frac{\partial^2 q}{\partial y^2}\right)+\left[\frac{1}{1+|q|^2}\right]q+pR(x,y)q=0 \qquad (1)$$

where x and y are transverse coordinates, and p is the modulation depth of the lattice. We suppose that optical lattice is created by a nth-order Bessel beam, whose field is given by the nth-order Bessel function $J_n[r]$, where $r^2=x^2+y^2$. Accordingly the lattice profile is determined by the beam intensity in the form of $R(x,y)=J_n^2[r]$. We search for the multi-poled soliton solutions of Eq. (1) numerically in the form of $q(x, y, z)=w(x, y)\exp(ibz)$. Here b is called propagation constant, and $w(x,y)$ is a real function. Substitution of this expression into Eq. (1) leads to

$$-bw+\frac{1}{2}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)+\frac{w}{1+w^2}+pJ_n^2[r]w=0 \qquad (2)$$

We solve Eq. (2) numerically using the finite difference method and the standard relaxation method. The initial iterative guess solution of $w(x,y)$ is written as

$$w(x,y)=\begin{cases}\cos(n\theta), & (r\le L)\\ 0, & (r>L)\end{cases} \qquad (3)$$

Here L is the first zero point of the nth-order Bessel function. Under this initial condition, the resulting solution is a $2n$-poled solitons. When changing the values of parameters p and b, the resulting intensity profiles of the n-poled solitons are shown in Fig. 1. Here, we take $n=7$, in which case $L=11.1$ for example, and the resulting soliton is a 14-poled soliton. It is shown that 14 bright dots reside homogeneously in the first ring of the 7th-order Bessel lattice.

Eq. (1) conserves the total power $U=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|q|^2dxdy=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}w^2dxdy$ as no gain or loss is considered in this model. Numerical results indicate that the power conveyed by the solitons are related with b and p. Energy spectrum is illustrated by Fig. 2 which shows the dependence of solitons power on propagation constant b and lattice depth p. The deeper the modulation depth p is, the higher the solitons power U is. In the other hand, soliton power decreases when the propagation constant b increases. This conclusion also can be comprehended from Fig. 1. When modulation parameter p keeps unchanged and propagation constant b decreases, the amplitude of the multi-poled lattice solitons increases while the solitons width hardly changes; the solitons increase its amplitude when the modulation depth p increases from 10 to 20 and b keeps unchanged. It should be pointed out that the asymptotic behavior of the U–b curve when $b\to 0$ is different from that in [13]. In [13], U diverges when $b\to 0$ as for multipole solitons in Bessel lattices with defocusing Kerr nonlinearity. In our paper, U converges when $b\to 0$ as for necklace solitons in the defocusing photorefractive media. The reason might be that the width of the incident beam that results in the multipole solitons in [13] is infinity. In this case, the soliton expands to more lattice rings (see Fig. 1(b) and (d) in [13]) besides that the soliton amplitude increases

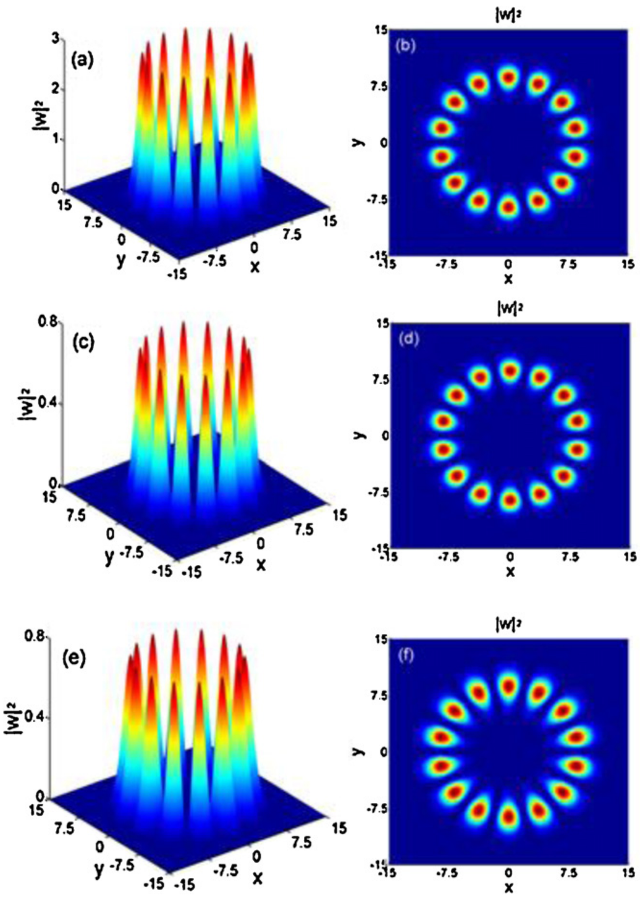

Fig. 1. Stationary solutions of 14-poled solitons. (a), (c), and (e): 3-D profiles; (b), (d) and (f): contour of optical intensity; (a) and (b): $p=20$ and $b=1$; (c) and (d): $p=20$ and $b=2$; (e) and (f): $p=10$ and $b=1$.

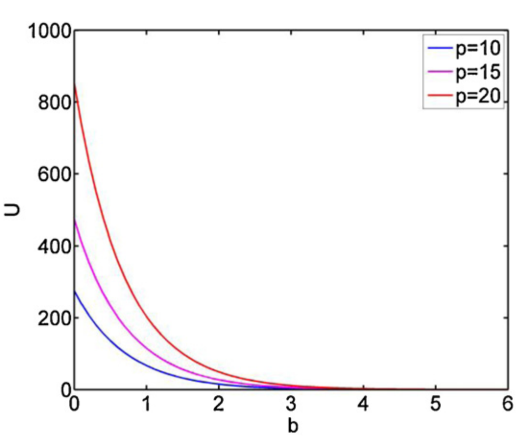

Fig. 2. Power U conveyed by the 14-poled solitons vs propagation constant b for different modulation depth p.