Optik 127 (2016) 10247–10258

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Original research article

Multi-switching combination synchronization of three different chaotic systems via nonlinear control

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ARTICLE INFO

Article history:
Received 7 June 2016
Accepted 8 August 2016

Keywords:
Chaotic system
Multi-switching synchronization
Combination synchronization
Nonlinear control

ABSTRACT

In this paper, the multi-switching combination synchronization among three different chaotic systems is investigated. Multi-switching combination synchronization of chaotic systems means that the state variables of two drive systems synchronize with different state variables of one response system, simultaneously. Here, the Lorenz system and Chen system are considered as the drive systems, the combination of the multi-drive systems is synchronized with Lü response system using multi-switching synchronization method. Based on nonlinear control technique and Lyapunov stability theory, sufficient conditions are obtained to achieve the desired synchronization among three different chaotic systems. For suitable choice of scaling factors, switching modified projective synchronization is obtained as a special case of multi-switching combination synchronization among different chaotic systems. Numerical simulations are shown to verify the feasibility and effectiveness of the proposed scheme.

1. Introduction

Chaos synchronization has become a hot subject in the field of nonlinear science since it was first investigated by Pecora and Carroll in 1990 [1]. Due to its great potential applications in physical systems, biological networks, information sciences and secure communications [2–5], etc. Up to now, many types of synchronization such that complete synchronization [6], phase synchronization [7], generalized synchronization [8], lag synchronization [9], projective synchronization [10], modified projective synchronization [11], function projective synchronization [12] and modified function projective synchronization [13,14], have been proposed and investigated in the past years. Meanwhile, various synchronization schemes

such as active control method [15], adaptive control method [16], sliding mode control [17], active backstepping method [18], impulsive control [19], pinning control [20], and so on, have been introduced and successfully applied to chaos synchronization problems.

Recently, new chaos synchronization ideas have been investigated wherein three or more chaotic systems are involved. Synchronization schemes such as combination synchronization [21], combination-combination synchronization [22,23], compound synchronization [24,25], compound-combination synchronization [26] has recently been presented. In order to improve the security of information transmission via chaotic synchronization, it may be required that in drive-response synchronization, different states of the response system are synchronized with desired state of the drive system in a multi-switching manner. This form of synchronization was proposed by Ucar et al. [27] based on active control technique. Despite

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http://dx.doi.org/10.1016/j.ijleo.2016.08.014
0030-4026/

its clear relevance to information security, only a few studies of this kind of synchronization have been reported [28–31]. To the best of our knowledge, all the work on multi-switching synchronization of this kind reported in the literature have related to single-drive/single −response system. More recently, a new synchronization, called multi-switching combination synchronization, was investigated in [32]. Compared with the combination synchronization and its variants, switched synchronization of chaotic systems in which a state variable of the drive systems synchronize with a different state variable of the response system is a promising type of synchronization as it provides greater security in secure communications. Therefore, it would be very instructive and significant to investigate multi-switching combination synchronization in non identical chaotic systems.

Inspired by the above discussions, in this paper, we intend to apply the nonlinear control technique to investigate the multi-switching combination synchronization problems of a class of chaotic systems. Based on the nonlinear control method, sufficient conditions for three different chaotic systems have been obtained by constructing two suitable drive chaotic systems and a controlled response chaotic system.

The rest of this paper is organized as follows. Section 2 gives some preliminaries. In Section 3, we analyze the multi-switching combination synchronization of two drive Lorenz and Chen with the Lü response system. The nonlinear control method is used to design the controllers and sufficient conditions to achieve desired synchronization is stated and proved. Examples are provided to illustrate the effectiveness of the obtained scheme in Section 4. Finally some conclusions are given in Section 5.

2. Problem description

In the section, we firstly design the scheme of multi-switching combination synchronization in our drive-response synchronization scheme with two drive systems and one response system. The two drive systems are, respectively, given as follows:

$$\dot{x} = f_1(x) \tag{1}$$

$$\dot{y} = f_2(y) \tag{2}$$

and the response system is described as follow:

$$\dot{z} = g_1(z) + u \tag{3}$$

where the state vectors of the systems (1)-(3) are $x = (x_1, x_2, \cdots, x_n)^T$, $y = (y_1, y_2, \cdots, y_n)^T$, $z = (z_1, z_2, \cdots, z_n)^T$, respectively. $f_1, f_2, g_1 : R^n \to R^n$ are three continuous nonlinear vector functions, $u = (u_1, u_2, \cdots, u_n)^T : R \times R \times \cdots \times R \to R^n$ is the controller of the response system (3) which will be designed, respectively.

Definition 1. [21]: For the drive systems (1) and (2) and the response system (3), if there exist three constant matrices $A, B, C \in R^{n\times n}$ and $C \neq 0$ such that

$$\lim_{t\to\infty}\|e\| = \lim_{t\to\infty}\|Ax + By - Cz\| = 0, \tag{4}$$

where $\|\cdot\|$ represents a vector norm induced by the matrix norm, the drive systems (1) and (2) are realized combination synchronization with the response system (3). Here, the constant matrices A, B, C are called the scaling matrices.

Remark 1. If $A = diag(\alpha_1, \alpha_2, \cdots, \alpha_n)$, $B = diag(\beta_1, \beta_2, \cdots, \beta_n)$, $C = diag(\gamma_1, \gamma_2, \cdots, \gamma_n)$, from definition 1, we can obtain the error vector

$$e_{ijk} = \alpha_i x_i + \beta_j y_j - \gamma_k z_k, \tag{5}$$

where the indices of the error vector are strictly chosen to satisfy $i = j = k$, $(i, j, k = 1, 2, \cdots, n)$.

Definition 2. [32]: The drive systems (1) and (2) and the response system (3) are said to obtain multi-switching combination synchronization, if the error states (5) are redefined such that

$$\lim_{t\to\infty}\|e_{ijk}\| = \lim_{t\to\infty}\|\alpha_i x_i + \beta_j y_j - \gamma_k z_k\| = 0, \tag{6}$$

and $i = j \neq k$ or $i = k \neq j$ or $j = k \neq i$ or $i \neq k \neq j$ or $i \neq j = k$, where $i, j, k = 1, 2, \cdots, n$ and $\|\cdot\|$ is vector norm.

3. Main results

In this section, we will realize the multi-switching combination synchronization among three different chaotic systems. The Lorenz system and Chen system are, respectively, described as the drive systems as follows:

$$\begin{cases} \dot{x}_1 = a_1(x_2 - x_1) \\ \dot{x}_2 = -b_1 x_1 - x_2 - x_1 x_3 \\ \dot{x}_3 = x_1 x_2 - c_1 x_3 \end{cases} \tag{7}$$