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Improved cost functions for blind source separation based on linear prediction

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ABSTRACT

Some improved cost functions based on linear prediction for blind source separation (BSS) are proposed in this letter. For uncorrelated sources, our analysis shows that each proposed cost functions is a generalized Rayleigh quotient, so the optimization can be converted to a generalized eigendecomposition of a corresponding matrix pencil. The proposed cost functions are legible and simulation experiments show that they have better performance than that in the literature.

1. Introduction

Blind source separation (BSS) is a powerful technique for signal processing and has found many applications in communications, biomedical signal processing, digital image processing, etc [1–3]. The task of BSS is to estimate the unknown underlying source signals from the observed signals without knowing the original sources and their mixture process [4–9]. Blind source extraction (BSE) aims at only estimating the sources of interest which are much fewer in number than the orignal sources, so BSE is more effective and has better performance than BSS. Besides, BSE can utilize some potential a priori knowledge which can improve and accelerate the extraction procedure [10–14]. An algorithm for BSE based on short-term and long-term linear prediction was proposed in Refs. [15] and [16], respectively. While an algorithm for BSS based on the canonical correlation analysis approach was proposed in Ref. [17]. In this paper we propose some more intelligible cost functions based on linear prediction for BSS.

2. System description

The linear instantaneous model for BSS is given by Ref. [1]

$$\boldsymbol{x}(\mathrm{n}) = \boldsymbol{As}(\mathrm{n}), \tag{1}$$

where $\mathbf{s}(n) = [s_1((n), s_2(n), \cdots, s_M(n)]$ are the original unknown source signals and $\mathbf{x}(n) = [x_1(n), x_2(n), \cdots, x_M(n)]$ are the observed mixed signals; and $\mathbf{A}$ is an unknown mixing matrix of full rank; n is the time index or sample point. Suppose the unmixing vector is $\mathbf{w}$. Then the separated signal is

$$y(n) = \mathbf{w}^T \mathbf{x}(n). \tag{2}$$

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The proposed cost function for BSE with a linear predictor in Ref. [15] is

$$J(\boldsymbol{w}) = \frac{E\left\{e^2(n)\right\}}{E\left\{y^2(n)\right\}}, \tag{3}$$

where $E\{\cdot\}$ denotes the statistical expectation operator and $e(n)$ denotes the instantaneous output error of the linear predictor. Suppose $\mathbf{b} = \left[b_1, b_2, \cdots, b_p\right]$ as the coefficient vector of the linear predictor, thus $e(n)$ is given by

$$e(n) = y(n) - \sum\nolimits_m b_m y(n-m). \tag{4}$$

The interpret for the cost function (3) is that the normalized mean square prediction error (MSPE) of a signal will not change with its power level and this normalized MSPE is normally different from those associated with the other signals. Notice that predictability is a fundamental property of a signal, and the normalized prediction error is an inherent characteristic of the signal. The aforementioned algorithm does not utilize any *a priori* knowledge and all of the source signals are simultaneously separated, so it is actually an algorithm for BSS rather than for BSE.

3. Improved cost functions for BSS based on linear prediction

The first proposed cost function for BSS is

$$J_1(\boldsymbol{w}) = \frac{E\left\{e_1^2(n)\right\}}{E\left\{\hat{s}^2(n)\right\}}, \tag{5}$$

where $\hat{s}(n)$ is a linearly predicted source signal from the separated source signal $y(n)$ and is given by

$$\hat{s}(n) = \sum\nolimits_m b_m y(n-m) \tag{6}$$

and $e_1(n)$ denotes the prediction error and is given by

$$e_1(n) = \hat{s}(n) - y(n). \tag{7}$$

J_1 is the ratio of the power of the prediction error to the power of the signal and $1/J_1$ is the signal-to-noise ratio (SNR). Here, the signal is predicted form the estimated signal since the source is unknown in BSS. The minimizing of J_1 or the maximizing of $1/J_1$ will hopefully output a source.

The second proposed cost function for BSS is

$$J_2(\boldsymbol{w}) = \frac{E\left\{\hat{s}(n)y(n)\right\}}{\sqrt{E\left\{\hat{s}^2(n)\right\}}\sqrt{E\left\{y^2(n)\right\}}}, \tag{8}$$

which is the normalized correlation coefficient between $\hat{s}(n)$ and $y(n)$. The minimizing of J_2 will also hopefully output a source. When the separation is excellent, $\hat{s}(n)$ should be a good estimate for a source signal and thus will nearly equal to the separated signal $y(n)$, viz, we have $\hat{s}(n) \approx y(n)$. Then, J_2 becomes

$$J_2(\boldsymbol{w}) = \frac{E\left\{\hat{s}(n)y(n)\right\}}{\sqrt{E\left\{\hat{s}^2(n)\right\}}\sqrt{E\left\{\hat{s}^2(n)\right\}}} = \frac{E\left\{\hat{s}(n)y(n)\right\}}{E\left\{\hat{s}^2(n)\right\}}. \tag{9}$$

The third proposed cost function for BSS similar with J_2 in (9) is

$$J_3(\boldsymbol{w}) = \frac{E\left\{\hat{s}(n)e_1(n)\right\}}{E\left\{\hat{s}^2(n)\right\}}. \tag{10}$$

Minimizing J_3 will make the correlation relationship between $\hat{s}(n)$ and $e_1(n)$ minimal. In other words, the minimization of J_3 will make $\hat{s}(n)$ and $e_1(n)$ nearly orthogonal, as is similar with the orthogonality principle in statistics and signal processing [18].

After some derivations, we will have a unifying form for the cost function (3) and the proposed cost functions (5), (9), and (10) as follows

$$J(\boldsymbol{w}) = \frac{\left(\boldsymbol{w}^T\boldsymbol{A}\right)\boldsymbol{\Sigma}_1\left(\boldsymbol{w}^T\boldsymbol{A}\right)^T}{\left(\boldsymbol{w}^T\boldsymbol{A}\right)\boldsymbol{\Sigma}_2\left(\boldsymbol{w}^T\boldsymbol{A}\right)^T} = \frac{\boldsymbol{w}^T\left(\boldsymbol{A}\boldsymbol{\Sigma}_1\boldsymbol{A}^T\right)\boldsymbol{w}}{\boldsymbol{w}^T\left(\boldsymbol{A}\boldsymbol{\Sigma}_2\boldsymbol{A}^T\right)\boldsymbol{w}}, \tag{11}$$

where $\boldsymbol{\Sigma}_1$ and $\boldsymbol{\Sigma}_2$ are some diagonal matrices when the source signals are uncorrelated with each other. J in (11) is a generalized Rayleigh quotient, hence its optimization can be converted into the generalized eigendecomposition of the matrix pencil $\left(\mathbf{A}\boldsymbol{\Sigma}_1\mathbf{A}^T, \mathbf{A}\boldsymbol{\Sigma}_2\mathbf{A}^T\right)$ [19]. Hence, the sufficient and necessary condition of the separability of the proposed cost