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Analysis of spectral properties of harmonic undulator radiation of an electromagnet undulator

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ABSTRACT

In this paper we analyze the spectral properties of undulator radiation with an electromagnet undulator for electron injected off the undulator axis. The electromagnet undulator is self bi-harmonic for locations near to the electromagnet. The electrons execute additional betatron oscillation when it injected off the undulator axis. It is observed that the electromagnet undulator can be used for harmonic lasing free electron laser operates at the cost of betatron oscillations for electron injected off the axis.

1. Introduction

There are interests in the study and design of micro-undulators for free electron laser and synchrotron radiation applications. A free electron laser with a micro-undulator required low electron beam energy and operate as a desktop system. In past, a number of micro-undulator designs based on electromagnet undulator have been studied and reported [1–8]. Works on pulsed ferromagnetic core electromagnet [9], tunable micro-undulators for free electron lasers are also studied [10]. The electromagnet wiggler from copper foil windings is reported. The design is based on a concept that combines two conducting foils in backward and forward directions [11]. An alternate design on electromagnet wigglers is presented [12]. The small period electromagnet undulator with iron free and thin wires is also reported [13]. The permanent magnet assisted electromagnet wiggler has been developed [14]. There are high current pulsed wire designs [20]. Electromagnet helical micro-wiggler [21,22] and staggered ferromagnetic core arrays are immersed in a sinusoidal field [23]. Harmonic undulator radiations with constant magnetic field are recently presented by Jeevakhan et al. [24].

In this paper we reconsider the theory of spectral properties of undulator radiation based on electromagnet micro-undulator [1–8]. In this schematic copper conductor of finite width runs alternatively between ferromagnetic lamination a core. The design studies of this electromagnet undulator are based on the calculation of the side leakage flux under the assumption of infinite permeability of the ferromagnetic lamination cores. Recently Huse et al. [8] have improved and alternate design of electromagnet undulator.

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In this paper we study the bi-harmonic properties of this electromagnet undulator for the two optimization model study by Radack et al. [4] and Huse et al. [8]. In the case of permanent bi-harmonic undulator schemes, one has to suitably locate the ferromagnetic shims in order to make it operate as bi-harmonic [15–19]. The bi-harmonic-undulators are useful for harmonic lasing of the free electron laser. The electromagnetic undulator is self bi-harmonic when electron beam is injected near to the electromagnet. Whenever the relativistic electron beam is injected off the axis, it gives rise to betatron oscillations and the intensity at the fundamental and its harmonics decrease. In Section 2 we develop the theory of bi-harmonic undulator and include the important effects of the betatron motion for off-axis electron beam injection. The results of the analysis are presented in Section 3.

2. Undulator radiation

In the air gap region of the electromagnet undulator, the magnetic field is expressed [1–8].

$$B_y(y,z) = -\mu_0 \sum_{k=1}^{\infty} \frac{(2k-1)2\pi}{\lambda_u} A_{2k-1} \sin\left(\frac{(2k-1)2\pi z}{\lambda_u}\right) \times \cosh\left(\frac{(2k-1)2\pi y}{\lambda_u}\right) \quad (1)$$

where

$$A_{2k-1} = \frac{I\lambda_u}{(2k-1)^2\pi^2 h} \left(\frac{\sin\left[(2k-1)\pi h/\lambda_u\right]}{\sinh[(2k-1)\pi\delta/\lambda_u]}\right) \quad (2)$$

where I is the current passing in copper plates, λ_u is the undulator period, δ is the air gap between two copper plates, h is the thickness of the copper plate and μ_0 is the permeability in free space. Rewriting $k = 1, 2$ terms in Eqs. (1) and (2) we get,

$$B_y(y,z) = -\mu_0 \frac{2\pi}{\lambda_u} A_1 \sin\left(\frac{2\pi z}{\lambda_u}\right)\cosh\left(\frac{2\pi y}{\lambda_u}\right) - 3\mu_0 \frac{2\pi}{\lambda_u} A_3 \sin\left(\frac{6\pi z}{\lambda_u}\right)\cosh\left(\frac{6\pi y}{\lambda_u}\right) \quad (3)$$

where

$$A_1 = \frac{I\lambda_u}{\pi^2 h}\frac{\sin(\pi h/\lambda_u)}{\sinh(\pi\delta/\lambda_u)}, \quad A_3 = \frac{I\lambda_u}{9\pi^2 h}\frac{\sin(3\pi h/\lambda_u)}{\sinh(3\pi\delta/\lambda_u)} \quad (4)$$

Using Eq. (4) in Eq. (3) we get,

$$B_y(y,z) = -\frac{2\mu_0 I}{\pi h}\frac{\sin(\pi h/\lambda_u)}{\sinh(\pi\delta/\lambda_u)}\sin\left(\frac{2\pi z}{\lambda_u}\right)\cosh\left(\frac{2\pi y}{\lambda_u}\right) - \frac{2\mu_0 I}{3\pi h}\frac{\sin(3\pi h/\lambda_u)}{\sinh(3\pi\delta/\lambda_u)}\sin\left(\frac{6\pi z}{\lambda_u}\right)\cosh\left(\frac{6\pi y}{\lambda_u}\right) \quad (5)$$

Denoting,

$$B_0 = \frac{2\mu_0 I}{h}$$

$$a_1 = \frac{\sin(\pi h/\lambda_u)}{\pi}\frac{1}{\sinh(\pi\delta/\lambda_u)}$$

$$a_3 = \frac{\sin(3\pi h/\lambda_u)}{3\pi}\frac{1}{\sinh(3\pi\delta/\lambda_u)}$$

Eq. (5) can be written as,

$$B_y(y,z) = -B_0 a_1\left[\sin(k_u z)\cosh(k_u y) + \frac{a_3}{a_1}\sin(3k_u z)\cosh(3k_u y)\right] \quad (6)$$

Eq. (6) is valid for regions away from the undulator axis i.e. $y \neq 0$.

The on-axis electron trajectories can be found from the Lorentz force equation for $y = 0$.

This is given by,

$$\beta_{x_0} = -\frac{K_1}{\gamma}\left[\cos(\Omega_u t) + \frac{K_3}{K_1}\cos(3\Omega_u t)\right]$$
$$x_0(t) = -\frac{K_1 c}{\gamma}\left[\frac{\sin(\Omega_u t)}{\Omega_u} + \frac{K_3}{K_1}\frac{\sin(3\Omega_u t)}{3\Omega_u}\right] \quad (7)$$

where

$$K_1 = \frac{eB_0 a_1\lambda_u}{2\pi mc^2}, \quad \frac{K_3}{K_1} = \frac{1}{3}\frac{a_3}{a_1} = \eta.$$

The electron axial velocity can be determined from conservation of energy i.e. $\gamma^2 = 1/(1-\beta^2)$ and we get,

$$\beta_z = \beta^* - \frac{K_1^2}{2\gamma^2}\left[\cos(2\Omega_u t) + \frac{1}{9}\frac{a_3^2}{a_1^2}\cos(6\Omega_u t)\right]$$
$$z = \beta^* ct - \frac{K_1^2 c}{2\gamma^2}\left[\frac{\cos(2\Omega_u t)}{2\Omega_u} + \frac{1}{9}\frac{a_3^2}{a_1^2}\frac{\cos(6\Omega_u t)}{6\Omega_u}\right] \quad (8)$$

where

$$\beta^* = 1 - \frac{1}{2\gamma^2} - \frac{1}{4\gamma^2}K_1^2\left(1 + \frac{1}{9}\frac{a_3^2}{a_1^2}\right)$$

where K_1 and K_3 define the undulator parameters of the respective fields and we have dropped the electron velocity modulation at $\Omega_u \pm 3\Omega_u$ from our analysis.

Eq. (6) represents the electromagnet undulator near the magnet surface which is away from the undulator axis i.e. $y \neq 0$. It has two field components. The first one is the oscillation at Ω_u and the second field component at the third harmonic $3\Omega_u$ To satisfy Maxwell equations, we rewrite Eq. (6) as.

$$B_y(y,z) = -B_0 a_1\left[\sin(k_u z)\cosh(k_u y) + \frac{a_3}{a_1}\sin(3k_u z)\cosh(3k_u y)\right]$$
$$B_z(y,z) = -B_0 a_1\left[\cos(k_u z)\sinh(k_u y) + \frac{a_3}{a_1}\cos(3k_u z)\sinh(3k_u y)\right] \quad (9)$$

With small argument expansion of the hyperbolic functions, Eq. (9) reads,

$$B_y = -B_0\left\{a_1\left(1 + \frac{k_u^2 y^2}{2}\right)\sin(k_u z) + a_3\left(1 + \frac{9k_u^2 y^2}{2}\right)\sin(3k_u z)\right\}$$
$$B_z = -B_0 k_u y(a_1\cos(k_u z) + 3a_3\cos(3k_u z)) \quad (10)$$

Now using Lorentz force equation and solve it we get.

$$\ddot{x} = \frac{eB_0\dot{z}}{m_0\gamma c}\left(a_1\left(\frac{k_u^2 y^2}{2}\right)\sin(k_u z) + a_3\left(\frac{9k_u^2 y^2}{2}\right)\sin(3k_u z)\right) - \frac{eB_0\dot{y}y}{m_0\gamma c}(a_1 k_u(\cos(k_u z) + 3a_3 k_u\cos(3k_u z))) \quad (11)$$
$$\ddot{y} = \frac{e}{m_0\gamma c}B_0\dot{x}y(k_u a_1\cos(k_u z) + 3a_3 k_u\cos(3k_u z))$$

We assume that the motion can be decomposed $x = x_0 + x_1$ and $y = y_0 + y_1$ where x_0 and y_0 are the reference trajectories for field represented at $y = 0$ and x_1 and y_1 additional motions arising due to additional off-axis field in Eq. (11). For a field specified in Eq. (6), $y = 0$ and x_0 is expressed in Eq. (7) extracting the additional betatron motion we find that [26],

$$\frac{d^2 x_1}{dt} = 0$$
$$\frac{d^2 y_1}{dt^2} + \Omega_\beta^2 y_1 = 0 \quad (12)$$

The betatron oscillations are described by,

$$\Omega_\beta^2 = \frac{{K_1}^2\Omega_u^2}{2\gamma^2}(1+\eta^2)$$

where

$$\eta = \frac{1}{3}\frac{a_3}{a_1}$$

The solution to Eq. (12) gives,

$$x_1(t) = x_1(0) + \dot{x}_1(0)t$$
$$y_1(t) = y_1(0)\cos(\Omega_\beta t) + \frac{\dot{y}_1(0)}{\Omega_1}\sin(\Omega_\beta t) \quad (13)$$

where $x_1(0)$ and $y_1(0)$ represent the off-axis positions from the undulator axis. $\dot{x}_1(0)$ and $\dot{y}_1(0)$ describes the electron injection angles. If injection angles $\dot{x}_1(0) = \dot{y}_1(0) = 0$, then solution

$$x_1(t) = x_1(0)$$
$$y_1(t) = y_1(0)\cos(\Omega_\beta t) \quad (14)$$

The electron trajectory is now described by,

$$\beta_x = -\frac{K_1}{\gamma}[\cos(k_u z) + \eta\cos(3k_u z)]$$

$$\beta_y = -\frac{y_1(0)}{c}\Omega_\beta\sin(\Omega_\beta t)$$

$$\beta_z = \beta^{**} - \frac{K_1^2}{4\gamma^2}[\cos(2k_u z) + \eta^2\cos(6k_u z)] + \frac{y_1^2(0)\Omega_\beta}{4c^2}\cos(2\Omega_\beta t)$$