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Focal region field expressions for a spherical reflector backed by chiral substrate

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ABSTRACT

High frequency field expression at the focal region of a spherical reflector antenna backed by isotropic and homogeneous chiral substrate is derived using Maslov's method. As Geometrical Optics (GO) solution fails at the focal point, so Maslov's method is used to overcome this problem. Field patterns are calculated numerically and the results are plotted. Analysis is done to show the effects of thickness (d) of chiral substrate, chirality parameter (β) of the chiral substrate and permittivity (ε) of the medium.

1. Introduction

In modern communication systems at very high frequency various type reflector are design and analyzed by different method. Among those method asymptotic ray theory (ART), or geometrical optics (GO) is well known method used for the analysis of general wave motion in various mediums associated with reflectors [1–3]. Mathematically in physical space, there exists singularity at the focal point but the singularity is not a real one. In fact, the solutions of electromagnetic wave equation is not singular therefore the GO approximation is not suitable for the caustic region where it predicts singularity. This singularity depends of the formulation. To avoids these problems and find field expressions at the focal regions an alternative method must be used. Therefore, an asymptotic method based on Maslov's theory [4,5] is used to study the behavior of field pattern around the focal regions. Maslov's method is used by many authors for various focusing systems as given in [6–17]. Maslov's method was originally developed for the purpose of extracting uniform asymptotic solutions of partial differential equations, such as the Schrodingers wave equation. According to Maslov's method, the ray is expressed in terms of hybrid coordinate components that is wave vector coordinate components $\mathbf{P} = (p_x, p_y, p_z)$ and space coordinate components $\mathbf{R} = (x, y, z)$. Maslov's method use the simplicity of ray theory and the generality of Fourier transform. The information of ray trajectory is included in both the coordinate systems, so the singularities in GO is avoided by using the mixed coordinate system. In this paper we use Maslov's method to derive the field expressions around the focal region of a spherical reflector backed by isotropic and homogeneous chiral substrate which is an extension of the two dimensional case.

Chiral medium is microscopically continuous medium composed of chiral objects, uniformly distributed and randomly oriented [18]. A chiral object is a three dimensional body that can not be brought into congruence with its mirror image through translation or rotation such as helix, animal hands, sugars, amino acids, and most other bio-molecules come in two forms (enantiomers) etc. Chiral objects always come in pairs, conceptually. An object which is not chiral is called achiral. A chiral medium is either right handed or left handed and shows optical activity i.e rotate the axis of the ray pass through the medium and a slight change in the speed of right circularly polarized (RCP) and left circularly polarized wave (LCP) wave as given in [17,18]. The historical background and electromagnetic chirality has been analyzed by many authors [19–24]. In Section 2, GO and Maslov's method in free space is given. In Section 3, the reflection coefficient of plane waves from a chiral substrate backed by perfect electric conductor is discussed. In Section 4, GO field for spherical reflector with chiral substrate is illustrated. In Section 5, results are plotted. Concluding remarks are given in Section 6.

2. Geometrical optics and Maslov's method in free space

The GO and Maslov's method have been used to analyze many focusing systems [6–17], but here it is applied to a spherical

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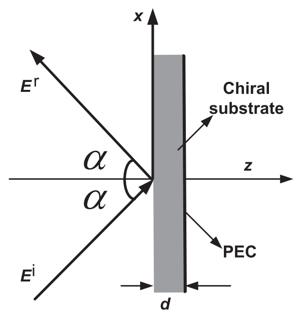

Fig. 1. Reflection of plane waves from chiral substrate backed by PEC plane.

reflector coated with chiral medium. consider a three dimensional wave equation

$$(\nabla^2 + k_0^2)u(r) = 0 \tag{1}$$

where $r = (x, y, z)$, $\nabla^2 = \partial^2/\partial_x^2 + \partial^2/\partial_y^2 + \partial^2/\partial_z^2$ and $k_0 = \omega\sqrt{\epsilon_0\mu_0}$ is wave number of the medium. Solution of the general wave equation (1) is assumed in the form of Luneberg–Kline series as

$$u(r) = \sum_{m=0}^{\infty} \frac{E^m(r)}{(jk_0)^m} \exp(-jk_0 s) \tag{2}$$

Assuming large values of k, in the above series, the higher order terms can be neglected, because they do not contribute and only the first term is retained. By putting Eq. (2) in Eq. (1) and equating the coefficients of k^2, the Eikonal equation is obtained. Eikonol equation is used to fine the phase of the wave as given by [25]

$$(\nabla s(r))^2 - 1 = 0 \tag{3}$$

For the amplitude of the wave we equate the coefficients of k, the transport equation is obtained

$$2\nabla E \cdot \nabla s + E\nabla^2 s = 0 \tag{4}$$

only E^0 has been retained and is denoted with E. Wave vector is define as $\mathbf{p} = \nabla s$ and Hamiltonian $H(r, p) = (\mathbf{p} \cdot \mathbf{p} - 1)/2$. So the eikonal equation becomes $H(r, p) = 0$ which describe the path of the ray. Hamiltonian equation in Cartesian coordinates are given as follows

$$\frac{dx}{dt} = p_x, \quad \frac{dy}{dt} = p_y, \quad \frac{dz}{dt} = p_z, \tag{5}$$

$$\frac{dp_x}{dt} = 0, \quad \frac{dp_y}{dt} = 0, \quad \frac{dp_z}{dt} = 0 \tag{6}$$

By solving these Hamiltonian equations we get

$$x = \xi + p_x t, \quad y = \eta + p_y t, \quad z = \zeta + p_z t \tag{7}$$

$$p_x = p_{x_0}, \quad p_y = p_{y_0}, \quad p_z = p_{z_0} \tag{8}$$

where (ξ, η, ζ) and (p_{x0}, p_{y0}, p_{z0}) are the initial values of (x, y, z) and (p_x, p_y, p_z) respectively. Moreover, t is the parameter along the ray. The phase function is given by

$$s = s_0 + \int_0^t dt = s_0 + t. \tag{9}$$

Applying Gauss's theorem to a paraxial ray tube, the solution of Eq. (4) is given by [25]

$$u(r) = E(r_0)J^{-1/2} \exp(-jk(s_0 + t)) \tag{10}$$

where $E(r_0)$ is the initial value of the field amplitude and $J = D(t)/D(0)$, where $D(t) = \partial(x, y, z)/\partial(\xi, \eta, \zeta)$ is the Jacobian of transformation from ray coordinates (ξ, η, ζ) to Cartesian coordinate (x, y, z). The GO solution is not valid at focal points that is where $J = 0$, so Maslov's method is used to find the fields around the caustic region of a focussing system as analyzed by [6–18]. The focal points can be avoided by mapping the rays to the wave vector domain. The mapping separates the rays that intersect at caustic. The Maslov's method uses the ray solution from one domain to correct the ray solution near caustics in the other domain. The Maslov's method is based upon Fourier transformation from spatial domain into spectral wave vector domain, or vice versa. The Fourier integral can be evaluated approximately by applying the stationary phase method. The stationary points are determined from the phase function. The equation which is valid around the focal point of a spherical reflector is given as [10]

$$u(r) = \frac{k}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(r_0) \left[\frac{D(t)}{D(0)} \frac{\partial(p_x, p_y)}{\partial(x, y)} \right]^{-\frac{1}{2}} \\ exp(-jk(s_0 + t - x(p_x, p_y, z)p_x - y(p_x, p_y, z)p_y + xp_x + yp_y))dp_x dp_y \tag{11}$$

The expression $\frac{D(t)}{D(0)} \frac{\partial(p_x, p_y)}{\partial(x, y)}$ can simply calculated as,

$$\frac{D(t)}{D(0)} \frac{\partial(p_x, p_y)}{\partial(x, y)} = \frac{1}{D(0)} \frac{\partial(p_x, p_y, z)}{\partial(\xi, \eta, \zeta)}. \tag{12}$$

3. Reflection of plane waves from a chiral substrate backed by PEC

In this section we want to find the reflected field around the focal region of a spherical reflector backed by chiral substrate. To achieve this the reflection of plane waves from a chiral slab backed by perfect electric conducting (PEC) plane is discussed as in [13]. As shown in Fig. 1 the region $z \leq 0$ is occupied by free space

$$\mathbf{D} = \epsilon_0 \mathbf{E}, \quad \mathbf{B} = \mu_0 \mathbf{H}$$

and the region $0 \leq z \leq d$ is occupied by the chiral medium. A chiral medium is a macroscopically continuous medium composed of equivalent chiral objects, uniformly distributed and randomly oriented in the host medium. A chiral object is a three dimensional body that cannot be brought into congruence with its mirror image through translation and rotation. The examples of chiral objects are abundant e.g. helix, animal hands, DNA molecules etc. An object which is not chiral is called achiral. A chiral object, and so is chiral medium, are either right handed or left handed. Chiral medium is analyzed by many researcher for different potential applications such as polarizer, antennas and other RF and photonics devices. Different techniques have been developed for analysis of wave propagation and boundary value problems involving chiral media which is defined by Drude–Born–Fadorov (DBF) constitutive relations [19] as follows

$$\mathbf{D} = \epsilon_0(\mathbf{E} + \beta\nabla \times \mathbf{E}), \quad \mathbf{B} = \mu_0(\mathbf{H} + \beta\nabla \times \mathbf{H})$$

where, ϵ is the permittivity and μ is the and permeability of the chiral substrate. β is the chirality parameter of the substrate. The incident electric field (E_i) and the reflected electric field (E_r) makes an angle α with the normal to the surface and can be expressed as

$$\mathbf{E}^i = \left\{ A_\perp \mathbf{a}_y + A_\parallel \left(-\frac{k_{0z}}{k_0}\mathbf{a}_x + \frac{k_{0x}}{k_0}\mathbf{a}_z \right) \right\} \times \exp(jk_{0z}z + jk_{0x}x) \tag{13}$$

and the reflected field as

$$\mathbf{E}^r = \left\{ B_\perp \mathbf{a}_y + B_\parallel \left(\frac{k_{0z}}{k_0}\mathbf{a}_x + \frac{k_{0x}}{k_0}\mathbf{a}_z \right) \right\} \times \exp(-jk_{0z}z + jk_{0x}x) \tag{14}$$

where, $A_\perp$, $B_\perp$ and $A_{||}$, $B_{||}$ are the perpendicular and parallel components w.r.t. the plane of incident respectively. $k_0 = \omega\sqrt{\epsilon_0\mu_0}$, $k_{0z} = k_0 \cos\alpha$ and $k_{0x} = k_0 \sin\alpha$. Field in the chiral layer can conveniently be written in terms of Beltrami fields [19] as

$$\mathbf{E} = \mathbf{Q}_L - j\eta\mathbf{Q}_R, \quad \mathbf{H} = \mathbf{Q}_R - j\mathbf{Q}_L/\eta \tag{15}$$