Optik 127 (2016) 1842–1847

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Effects of biological tissues on the propagation properties of anomalous hollow beams

Xingyuan Lu [a], Xinlei Zhu [a], Kuilong Wang [b], Chengliang Zhao [a,*], Yangjian Cai [a]

[a] College of Physics, Optoelectronics and Energy & Collaborative Innovation Center of Suzhou Nano Science and Technology, Soochow University, Suzhou 215006, China
[b] Department of Physics, Hangzhou Normal University, Hangzhou 310036, China

ARTICLE INFO

Article history:
Received 8 April 2015
Accepted 9 November 2015

Keywords:
Anomalous hollow beam
Biological tissues
Statistical properties

ABSTRACT

We derived analytical formulae of anomalous hollow beams (AHBs) passing through the turbulent biological tissues based on the extended Huygens–Fresnel integral formula. With the help of these formulae, we investigate the propagation properties of AHBs in turbulent biological tissues, the irradiance and spreading properties of AHBs in turbulent biological tissues are studied numerically. It is found that the circular and elliptical AHBs eventually become Gaussian beams in the far field and the central irradiance of the AHB rises more rapidly as the value of C_n^2 grows. We also calculate the formulae of the effective beam size of AHB and find that finally W_{xz} becomes equal to W_{yz} in turbulent biological tissues which can be used to explain the beam spot eventually becomes circular under the influence of turbulence of biological tissues.

1. Introduction

Dark hollow beams with zero central intensity have attracted much attentions due to its wide applications in atomic optics, free space optical communications, binary optics, optical trapping of particles and medical sciences [1–3]. There are many kinds of hollow beams, such as Hermite–Gauss beams, Laguerre–Gauss beams, Hollow Gaussian beams, Bessel–Gauss beams, Bessel-like beams, and so on, while one of them is very different from others which is named as anomalous hollow beam (AHB). The main difference between conventional dark hollow beam and AHB is that there is an elliptical solid core at the beam center of AHB. In 2005, Wu et al. first observed an anomalous hollow electron beam in an experiment [4], which can be used for studying the transverse instability and provide a powerful tool for studying the linear and nonlinear particle dynamics in the storage ring. In 2007, Cai proposed a theoretical model to describe an AHB [5]. After that, the propagation properties of AHB in free space [6] and uniaxial crystals [7] are studied. Although there are a lot of researches about AHBs [8–12], the exploration is still far from complete.

Recently, with the development of molecular biology and medicine, there is an urgent need to understand the living tissues, such as the changes during the process of development of embryos and the physiological and pathological changing processes of animal tissues. Due to the light of the abroad important applications, great efforts are addicted to the beams propagating through tissues [13–16]. In 1996, Schmitt and Kumar found that the structure function of refractive-index inhomogeneities in mammalian tissues fits the classical Kolmogorov model of turbulence [17].

In this paper, we study the propagation of an AHB passing through turbulent biological tissues. We derive analytical formulae of AHB in turbulent biological tissues. Some numerical examples are given to illustrate the propagation properties of an AHB in biological tissues.

2. Theory

The electric field of an AHB of elliptical symmetry at $z=0$ can be expressed as superposition of astigmatic Gaussian modes and astigmatic doughnut modes as follows [5]:

$$E(x,y,0)=\left(-2+\frac{8x^2}{w_{0x}^2}+\frac{8y^2}{w_{0y}^2}\right)\quad\exp\left(-\frac{x^2}{w_{0x}^2}-\frac{y^2}{w_{0y}^2}\right)\tag{1}$$

where w_{0x} and w_{0y} are the beam waist widths of an astigmatic Gaussian mode in x and y directions, respectively. When $w_{0x}=w_{0y}$, Eq. (1) reduces to a circular AHB.

* Corresponding author. Tel.: +86 512 69153532.
E-mail address: zhaochengliang@suda.edu.cn (C. Zhao).

http://dx.doi.org/10.1016/j.ijleo.2015.11.039
0030-4026/

Based on the extended Huygens–Fresnel integral formula, the propagation of a laser beam in the turbulent biological tissues can be described as [18–21].

$$\Gamma(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)=\frac{k^2}{4\pi^2z^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}E(\mathbf{r}_1,0)\ \ E^*(\mathbf{r}_2,0)\times\exp\left[-\frac{ik}{2z}(\mathbf{r}_1-\boldsymbol{\rho}_1)^2+\frac{ik}{2z}(\mathbf{r}_2-\boldsymbol{\rho}_2)^2\right]\times\langle\exp[\psi(\mathbf{r}_1,\boldsymbol{\rho}_1)+\psi^*(\mathbf{r}_2,\boldsymbol{\rho}_2)]\rangle d\mathbf{r}_1d\mathbf{r}_2 \quad (2)$$

with

$$\langle\exp[\psi(\mathbf{r}_1,\boldsymbol{\rho}_1)+\psi^*(\mathbf{r}_2,\boldsymbol{\rho}_2)]\rangle=\exp[-0.5\ \ D_\psi(\mathbf{r}_1-\mathbf{r}_2)]=\exp\left\{-\frac{1}{\rho_0^2}[(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)^2+(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)(\mathbf{r}_1-\mathbf{r}_2)+(\mathbf{r}_1-\mathbf{r}_2)^2]\right\} \quad (3)$$

where

$$|\rho_0|=0.22(C_n^2k^2z)^{-1/2},\quad C_n^2=\frac{\langle\delta n^2\rangle}{L_0(2-\varsigma)} \quad (4)$$

In Eq. (3), $\boldsymbol{r}=(x,y)$ and $\boldsymbol{\rho}=(\rho_x,\rho_y)$ are the position vectors at the input plane ($z=0$) and output plane (z), respectively $\Gamma(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)$ is the second-order correlation at the output plane (z), $E(r_1,0)$ is the electric field of the laser beam at the source plane ($z=0$), and $d\boldsymbol{r}_1d\boldsymbol{r}_2=dx_1dy_1dx_2dy_2$. In Eq. (4), C_n^2 is the structure constant of the refractive-index of the biological tissues. L_0 is the outer scale of the refractive-index size. ς is related to the fractal dimension of the tissue which is an indication of the classical turbulent behavior of biological tissue. $\langle\delta n^2\rangle$ is the ensemble-averaged variance of the refractive index.

Substituting Eq. (1) into Eq. (2), after tedious but straightforward integration, we obtain the following expression for the second-order correlation of AHB passing through the biological tissues:

$$\Gamma(\rho_{1x},\rho_{2x},\rho_{1y},\rho_{2y},z)=\exp\left[-\frac{ik}{2z}(\rho_{1x}^2+\rho_{1y}^2-\rho_{2x}^2-\rho_{2y}^2)-\frac{(\rho_{1x}-\rho_{2x})^2+(\rho_{1y}-\rho_{2y})^2}{\rho_0^2}\right]\times\exp\left[-\frac{k^2}{4b_xz^2}\left(\rho_{1x}-\frac{\rho_{2x}}{a_x\rho_0^2}\right)^2-\frac{k^2}{4b_yz^2}\left(\rho_{1y}-\frac{\rho_{2y}}{a_y\rho_0^2}\right)^2\right]\times\exp\left[-\frac{k^2}{4a_xz^2}\rho_{2x}^2-\frac{k^2}{4a_yz^2}\rho_{2y}^2\right]\exp[H(\rho_x)+H(\rho_y)]\times(\Gamma_1-\Gamma_2-\Gamma_3-\Gamma_4-\Gamma_5+\Gamma_6+\Gamma_7+\Gamma_8+\Gamma_9)] \quad (5)$$

where

$$\Gamma_1=\frac{k^2}{z^2\sqrt{a_xa_yb_xb_y}} \quad (6)$$

$$\Gamma_2=\frac{k^2}{z^2w_{0x}^2b_x{}^{5/2}\sqrt{a_xa_yb_y}}\left[2b_x-\frac{k^2}{z^2}\left(\rho_{1x}-\frac{\rho_{2x}}{a_x\rho_0^2}\right)^2+H_2(\rho_x)\right] \quad (7)$$

$$\Gamma_3=\frac{k^2}{z^2w_{0x}^2b_x^{*5/2}\sqrt{a_x^*a_yb_y}}\left[2b_x^*-\frac{k^2}{z^2}\left(\rho_{2x}-\frac{\rho_{1x}}{a_x^*\rho_0^2}\right)^2+H_3(\rho_x)\right] \quad (8)$$

$$\Gamma_4=\frac{k^2}{z^2w_{0y}^2b_y{}^{5/2}\sqrt{a_ya_xb_x}}\left[2b_y-\frac{k^2}{z^2}\left(\rho_{1y}-\frac{\rho_{2y}}{a_y\rho_0^2}\right)^2+H_2(\rho_y)\right] \quad (9)$$

$$\Gamma_5=\frac{k^2}{z^2w_{0y}^2b_y^{*5/2}\sqrt{a_y^*a_xb_x}}\left[2b_y^*-\frac{k^2}{z^2}\left(\rho_{2y}-\frac{\rho_{1y}}{a_y^*\rho_0^2}\right)^2+H_3(\rho_y)\right] \quad (10)$$

$$\Gamma_6=\frac{k^2}{z^2w_{0x}^4a_x{}^{5/2}\sqrt{a_yb_y}}(A_{1x}+A_{2x}+A_{3x}) \quad (11)$$

$$\Gamma_7=\frac{k^2}{z^2w_{0y}^4a_y{}^{5/2}\sqrt{a_xb_x}}(A_{1y}+A_{2y}+A_{3y}) \quad (12)$$

$$\Gamma_8=\frac{k^2}{z^2w_{0x}^2w_{0y}^2b_x^{*5/2}b_y{}^{5/2}\sqrt{a_x^*a_y}}\times\left[2b_x^*-\frac{k^2}{z^2}\left(\rho_{2x}-\frac{\rho_{1x}}{a_x^*\rho_0^2}\right)^2+H_3(\rho_x)\right]\times\left[2b_y-\frac{k^2}{z^2}\left(\rho_{1y}-\frac{\rho_{2y}}{a_y\rho_0^2}\right)^2+H_2(\rho_y)\right] \quad (13)$$

$$\Gamma_9=\frac{k^2}{z^2w_{0x}^2w_{0y}^2b_y^{*5/2}b_x{}^{5/2}\sqrt{a_y^*a_x}}\times\left[2b_y^*-\frac{k^2}{z^2}\left(\rho_{2y}-\frac{\rho_{1y}}{a_y^*\rho_0^2}\right)^2+H_3(\rho_y)\right]\times\left[2b_x-\frac{k^2}{z^2}\left(\rho_{1x}-\frac{\rho_{2x}}{a_x\rho_0^2}\right)^2+H_2(\rho_x)\right] \quad (14)$$

with

$$a_x=\frac{1}{w_{0x}^2}-\frac{ik}{2z}+\frac{1}{\rho_0^2},\quad a_x^*=\frac{1}{w_{0x}^2}+\frac{ik}{2z}+\frac{1}{\rho_0^2}$$

$$a_y=\frac{1}{w_{0y}^2}-\frac{ik}{2z}+\frac{1}{\rho_0^2},\quad a_y^*=\frac{1}{w_{0y}^2}+\frac{ik}{2z}+\frac{1}{\rho_0^2}$$

$$b_x=a_x^*-\frac{1}{a_x\rho_0^4},\ b_x^*=a_x-\frac{1}{a_x^*\rho_0^4}$$

$$b_y=a_y^*-\frac{1}{a_y\rho_0^4},\ b_y^*=a_y-\frac{1}{a_y^*\rho_0^4}$$

$$\rho_x=\rho_{2x}-\rho_{1x},\ \rho_y=\rho_{2y}-\rho_{1y}$$

$$A_{1x}=\frac{1}{\rho_0^4b_x^{9/2}}\left[12b_x{}^2-12b_x\frac{k^2}{z^2}\left(\rho_{1x}-\frac{\rho_{2x}}{a_x\rho_0^2}-H_0(\rho_x)\right)^2+\frac{k^4}{z^4}\left(\rho_{1x}-\frac{\rho_{2x}}{a_x\rho_0^2}-H_0(\rho_x)\right)^4\right]$$

$$A_{2x}=-\frac{2\left(-k^2\rho_0^2\rho_{2x}+ikz\rho_x\right)}{z^2\rho_0^4b_x^{7/2}}\left(\rho_{1x}-\frac{\rho_{2x}}{a_x\rho_0^2}-H_0(\rho_x)\right)\times\left[6b_x-\frac{k^2}{z^2}\left(\rho_{1x}-\frac{\rho_{2x}}{a_x\rho_0^2}-H_0(\rho_x)\right)^2\right]$$